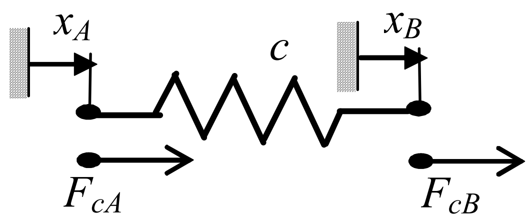
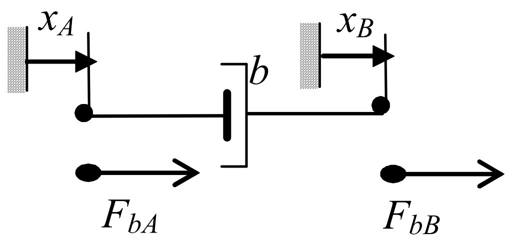
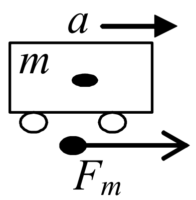
Proste układy mechaniczne

Założenie – jeden kierunek działania sił

1) Opis działania układu za pomocą idealnych elementów







$$F_{cA}(t) = c\left(x_A(t) - x_B(t)\right)$$

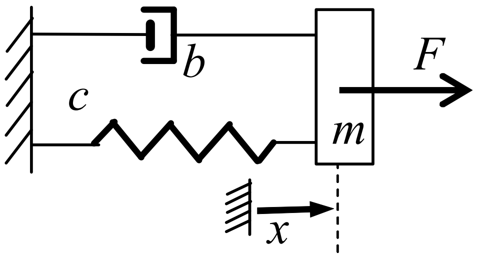
$$F_{cB}(t) = c\left(x_B(t) - x_A(t)\right)$$

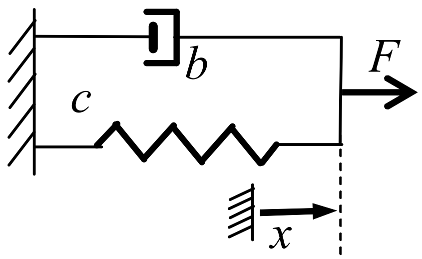
$$F_{bA}(t) = b\left(\dot{x}_A(t) - \dot{x}_B(t)\right)$$

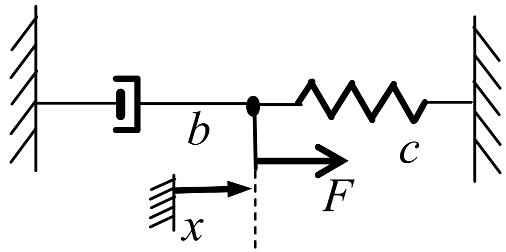
$$F_{bB}(t) = b\left(\dot{x}_B(t) - \dot{x}_A(t)\right)$$

$$F_m(t) = m\ddot{x}(t)$$

2) Punkt bilansowania sił





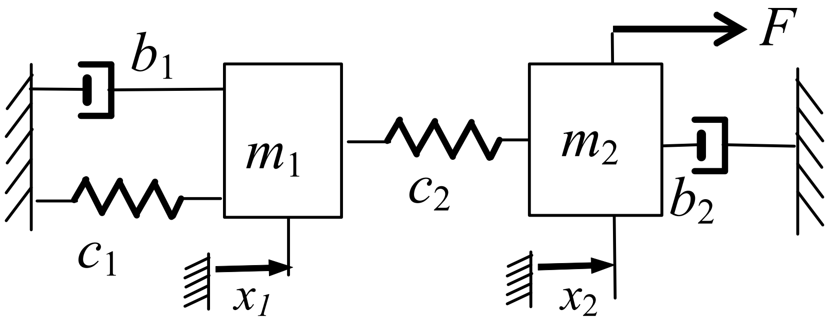


3) Bilans sił [N]

$$m\ddot{x}(t) + b\dot{x}(t) + cx(t) = F(t)$$

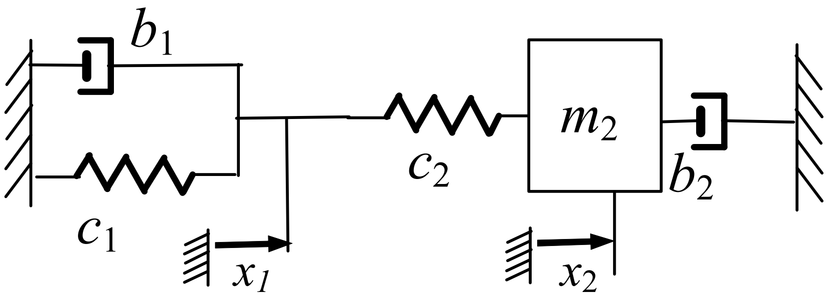
$$b\dot{x}(t) + cx(t) = F(t)$$

$$b\dot{x}(t) + cx(t) = F(t)$$



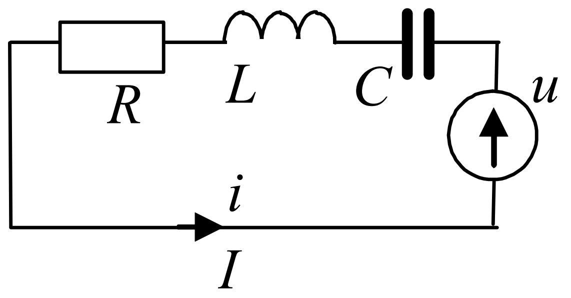
$$\begin{cases} F = m_2\ddot{x}_2 + b_2\dot{x}_2 + c_2(x_2 - x_1) \\ 0 = m_1\ddot{x}_1 + b_1\dot{x}_1 + c_1x_1 + c_2(x_1 - x_2) \end{cases}$$

(2 punkty, 2 masy)



$$\begin{cases} 0 = m_2\ddot{x}_2 + b_2\dot{x}_2 + c_2(x_2 - x_1) \\ 0 = b_1\dot{x}_1 + c_1x_1 + c_2(x_1 - x_2) \end{cases}$$

(2 punkty, 2 masy, bez zewnętrzbej siły)



(1) $$j\omega L\, I + R\, I + \frac{1}{j\omega C} I = U$$

(2) $$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)dt = u(t)$$

(3) $$L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C} q(t) = u(t)$$

(4) $$sL\, i(s) + R\, i(s) + \frac{1}{sC} i(s) = u(s)$$

(5) $$i(s) = \frac{sC}{s^2 LC + sRC + 1} u(s)$$

$$u(t) = U\sin(\omega t)$$

$$i(t) = I\sin(\omega t + \varphi)$$

$$s = j\omega$$

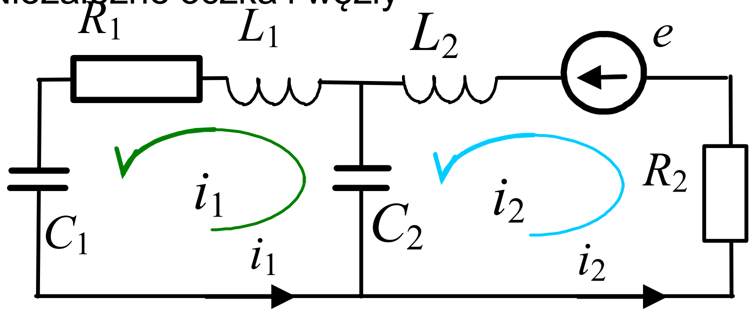
$$i(t) = \frac{dq(t)}{dt}$$

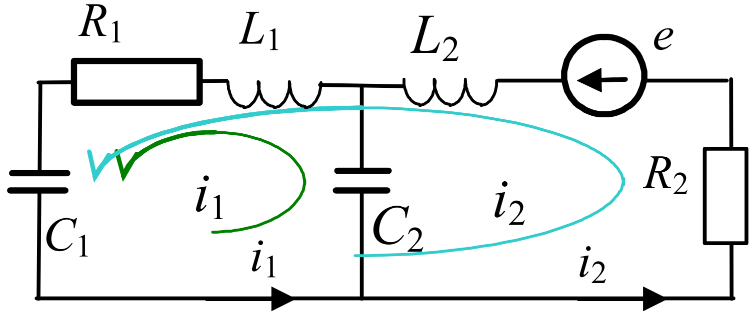
$$i(s) = sq(s)$$

1) Opis działania układu za pomocą idealnych elementów

	Opis napięciowo-prądowy $u(i)$		O.prąd.-napięciowy $i(u)$	$u(q)$	Impedancje	
					$Z(s)$	$Z(j\omega)$
rezystor (R)	$u(t)=Ri(t)$	$u(s)=Ri(s)$	$i(t)=Gu(t)$	$u(t)=R\dot{q}(t)$	R	R
kondensator (C)	$u(t)=\frac{1}{C}\int i(t)dt$	$u(s)=\frac{1}{sC}i(s)$	$i(t)=C\frac{du(t)}{dt}$	$u(t)=\frac{1}{C}q(t)$	$\frac{1}{sC}$	$\frac{1}{j\omega C}$
cewka (L)	$e_L(t)=-L\frac{di(t)}{dt}$	$u(s)=sLi(s)$	$i(t)=\frac{1}{L}\int u(t)dt$	$u(t)=L\ddot{q}(t)$	sL	$j\omega L$

2) Niezależne oczka i węzły





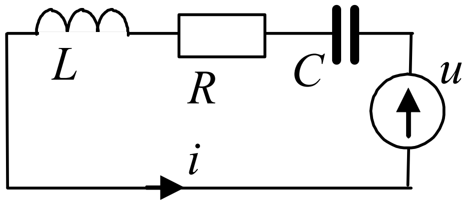
3) Bilans napięć w oczkach obwodu i/lub prądów (ładunków) w węzach [V, A]

$$\begin{cases} e = sL_2 i_2 + R_2 i_2(s) + \dfrac{i_2 - i_1}{sC_2} \\ 0 = sL_1 i_1 + R_1 i_1 + \dfrac{i_1}{sC_1} + \dfrac{i_1 - i_2}{sC_2} \end{cases}$$

$$\begin{cases} e = L_2 \dfrac{di_2}{dt} + R_2 i_2 + \int \dfrac{i_2 - i_1}{C_2} dt \\ 0 = L_1 \dfrac{di_1}{dt} + R_1 i_1 + \int \dfrac{i_1}{C_1} dt + \int \dfrac{i_1 - i_2}{C_2} dt \end{cases}$$

$$\begin{cases} e = L_2 \ddot{q}_2 + R_2 \dot{q}_2 + \dfrac{q_2 - q_1}{C_2} \\ 0 = L_1 \ddot{q}_1 + R_1 \dot{q}_1 + \dfrac{q_1}{C_1} + \dfrac{q_1 - q_2}{C_2} \end{cases}$$

Układy elektryczne i mechaniczne

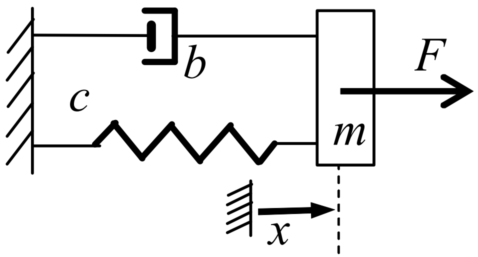


$$j\omega LI + RI + \frac{1}{j\omega C} I = U$$

$$sLi(s) + Ri(s) + \frac{1}{sC} i(s) = u(s)$$

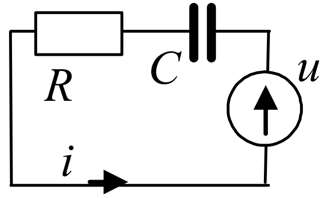
$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)dt = u(t)$$

$$L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C} q(t) = u(t)$$



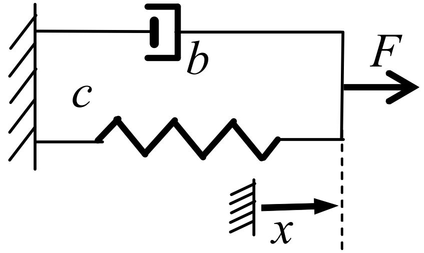
$$m\ddot{x}(t) + b\dot{x}(t) + cx(t) = F(t)$$

$$ms^2x(s) + bsx(s) + cx(s) = F(s)$$



$$R\dot{q}(t) + \frac{1}{C} q(t) = u(t)$$

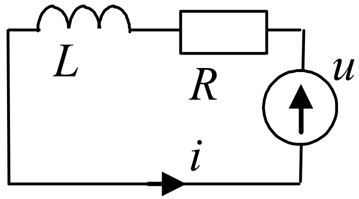
$$q(s) = \left(\frac{1}{sR + 1/C}\right) u(s)$$



$$b\dot{x}(t) + cx(t) = F(t)$$

$$bsx(s) + cx(s) = F(s)$$

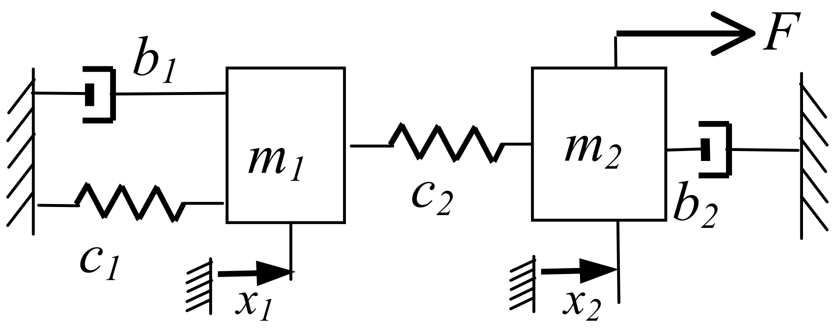
$$x(s) = \left(\frac{1}{sb + c}\right) F(s)$$

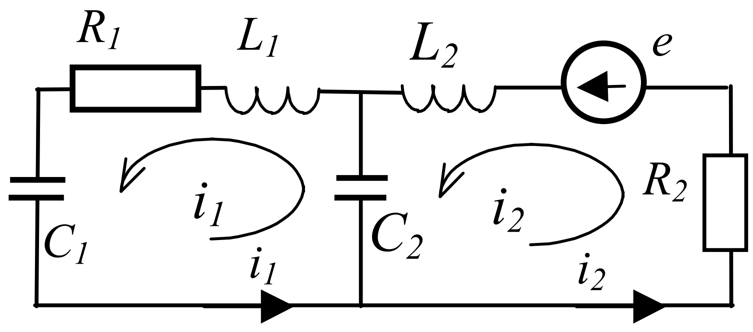


$$sLi(s) + Ri(s) = u(s)$$

$$i(s) = \left(\frac{1}{sL + R}\right) u(s)$$

Analogia układów mechanicznych i elektrycznych





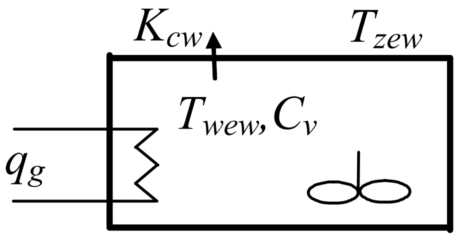
$$\begin{cases} F = m_2\ddot{x}_2 + b_2\dot{x}_2 + c_2(x_2 - x_1) \\ 0 = m_1\ddot{x}_1 + b_1\dot{x}_1 + c_1x_1 + c_2(x_1 - x_2) \end{cases}$$

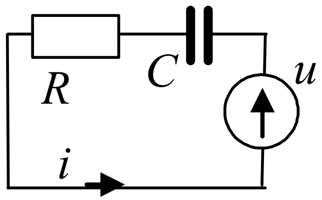
$$\begin{cases} F = s^2 m_2 x_2 + s b_2 x_2 + c_2(x_2 - x_1) \\ 0 = s^2 m_1 x_1 + s b_1 x_1 + c_1 x_1 + c_2(x_1 - x_2) \end{cases}$$

$$\begin{cases} e = sL_2 i_2 + R_2 i_2(s) + \dfrac{i_2 - i_1}{sC_2} \\ 0 = sL_1 i_1 + R_1 i_1 + \dfrac{i_1}{sC_{1\;1}} + \dfrac{i_1 - i_2}{sC_2} \end{cases}$$

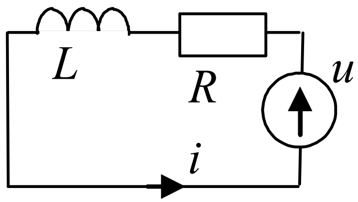
$$\begin{cases} e = L_2\ddot{q}_2 + R_2\dot{q}_2 + \dfrac{q_2 - q_1}{C_2} \\ 0 = L_1\ddot{q}_1 + R_1\dot{q}_1 + \dfrac{q_1}{C_1} + \dfrac{q_1 - q_2}{C_2} \end{cases}$$

Analogia cieplno-elektryczna





$$q(s)=\left(\frac{1}{sR+1/C}\right)u(s)$$

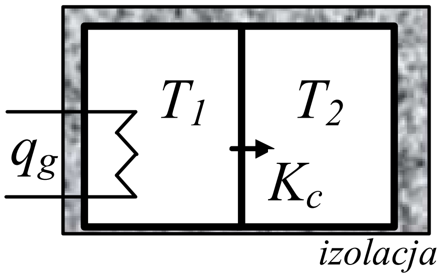


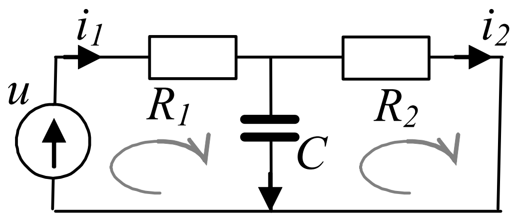
$$i(s)=\left(\frac{1}{sL+R}\right)u(s)$$

$$C_v\dot{T}_{wew}(t)=q_g(t)-K_c\left(T_{wew}(t)-T_{zew}(t)\right)$$

$$C_v sT_{wew}(s)+K_cT_{wew}(s)=q_g(s)+K_cT_{zew}(s)$$

$$T_{wew}(s)+T_{wew}(s)=\frac{1}{C_vs+K_c}q_g(s)+\frac{K_c}{C_vs+K_c}T_{zew}(s)$$





$$\begin{cases}C_{v1}\dot{T}_1(t)=q_g(t)-K_c\left(T_1(t)-T_2(t)\right)\\ C_{v2}\dot{T}_2(t)=K_c\left(T_1(t)-T_2(t)\right)\end{cases}$$

$$\begin{cases}R_1\dot{q}_1(t)+\dfrac{1}{C}\left(q_1(t)-q_2(t)\right)=u(t)\\ R_2\dot{q}_2(t)+\dfrac{1}{C}\left(q_2(t)-q_1(t)\right)=0\end{cases}$$

Analogie układów cieplnych, elektrycznych, mechanicznych, hydraulicznych

Tab.I-7. Przykłady analogii

	obiekty cieplne	obwody elektryczne			układy mechaniczne		układy hydrauliczne		
							zamknięte		otwarte
magazyn	$Q = C_V T$	$q = Cu$		$u = \frac{1}{C} q$		$F = cx$			$V = Ah$
	$\frac{dQ}{dt} = C_V \frac{dT}{dt}$	$\frac{dq}{dt} = C \frac{du}{dt}$							$\frac{dV}{dt} = A \frac{dh}{dt}$
	$q = C_V \frac{dT}{dt}$	$i = C \frac{du}{dt}$	$u = \frac{1}{C} \int i dt$		$F = c \int v dt$		$\Delta p = \frac{K}{V} \int f dt$	$f = \frac{V}{K} \frac{d\Delta p}{dt}$	
przewodność (lub opór)	$q = K_c T$	$i = \frac{1}{R} u$	$u = Ri$	$u = R \frac{dq}{dt}$	$F = bv$	$F = b \frac{dx}{dt}$	$\Delta p \approx Rf$ (*1)	$f = \frac{1}{R} \Delta p$	$f \approx ah$ (*2)
bezwładność		$i = \frac{1}{L} \int u dt$	$u = L \frac{di}{dt}$	$u = L \frac{d^2 q}{dt^2}$	$F = m \frac{dv}{dt}$	$F = m \frac{d^2 x}{dt^2}$	$\Delta p = m \frac{df}{dt}$		
źródła	q	i	u		F		Δp	f	f
funkcje czasu	$Q(t), q(t), T(t)$	$q(t), i(t), u(t)$			$x(t), v(t), F(t)$		$\Delta p(t), f(t)$		$V(t), f(t), h(t)$
bilans	$\sum q$	$\sum i$, $\sum u$			$\sum F$		$\sum q$, $\sum f$		$\sum f$

Uwaga: W tabeli pomięto oznaczenie funkcji czasu, na przykład jest T zamiast $T(t)$

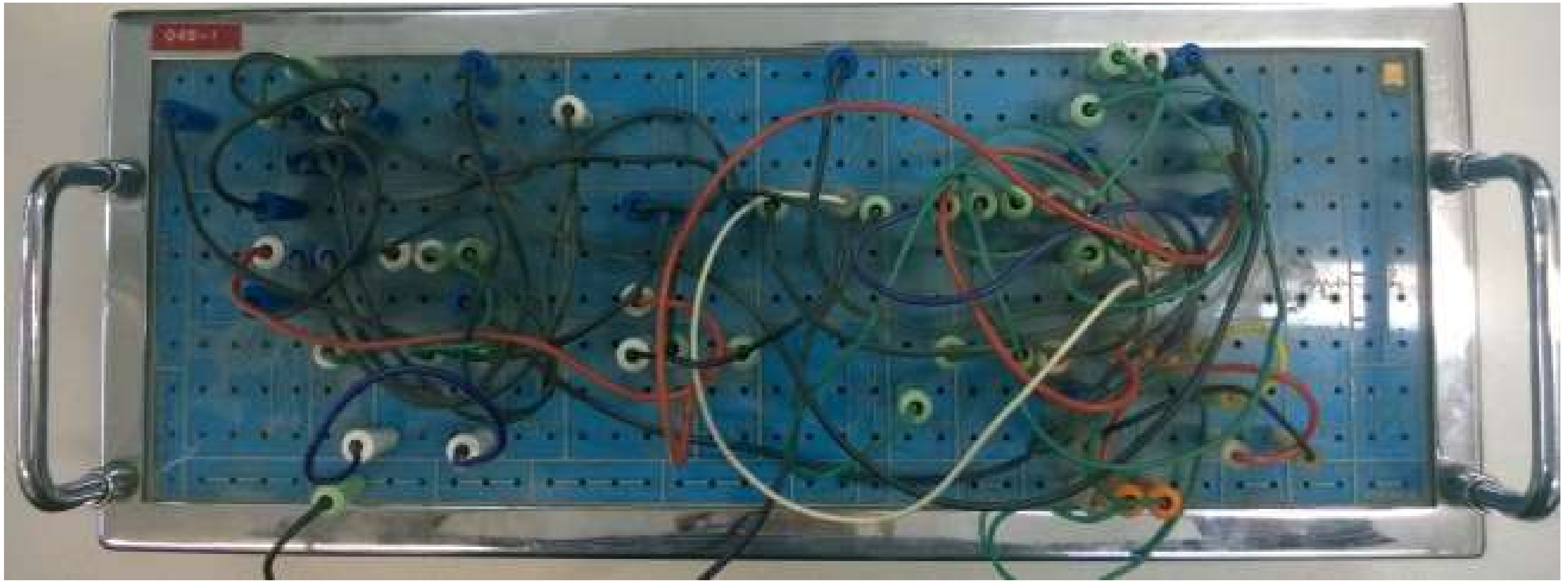
Analogie dotyczą opisu liniowego, natomiast zależności dokładne to: (*1) $\Delta p = Rf^2$, (*2) $f = k\sqrt{h}$

Zmienne i jednostki:

- obiekty cieplne: $Q(t)$ – ciepło [J], $q(t)$ – moc, strumień ciepła [W], $T(t)$ – temperatura [K],[°C];
- obwody elektryczne: $q(t)$ – ładunek elektryczny [C], $i(t)$ – natężenie prądu [A], $u(t)$ – napięcie, różnica potencjałów [V];
- układy mechaniczne: $x(t)$ – przesunięcie [m], $v(t)$ – prędkość [m/s], $F(t)$ – siła [N];
- układy hydrauliczne: $V(t)$ – objętość [m^3], $f(t)$ – przepływ, strumień [m^3/s], $\Delta p(t)$ – ciśnienie [Pa=N/m^2].

Modele analogowe

Maszyna analogowa (analogowa maszyna licząca)



Analogowa maszyna licząca (maszyna/komputer analogowy)

Zastosowania analogii

Przeniesienie wiedzy (doświadczenia, metod)

- uwaga na ograniczenia (zakres analogii)
- analogia – podobnieństwo (nie identyczność)

Uogólnienie

- teoria sterowania