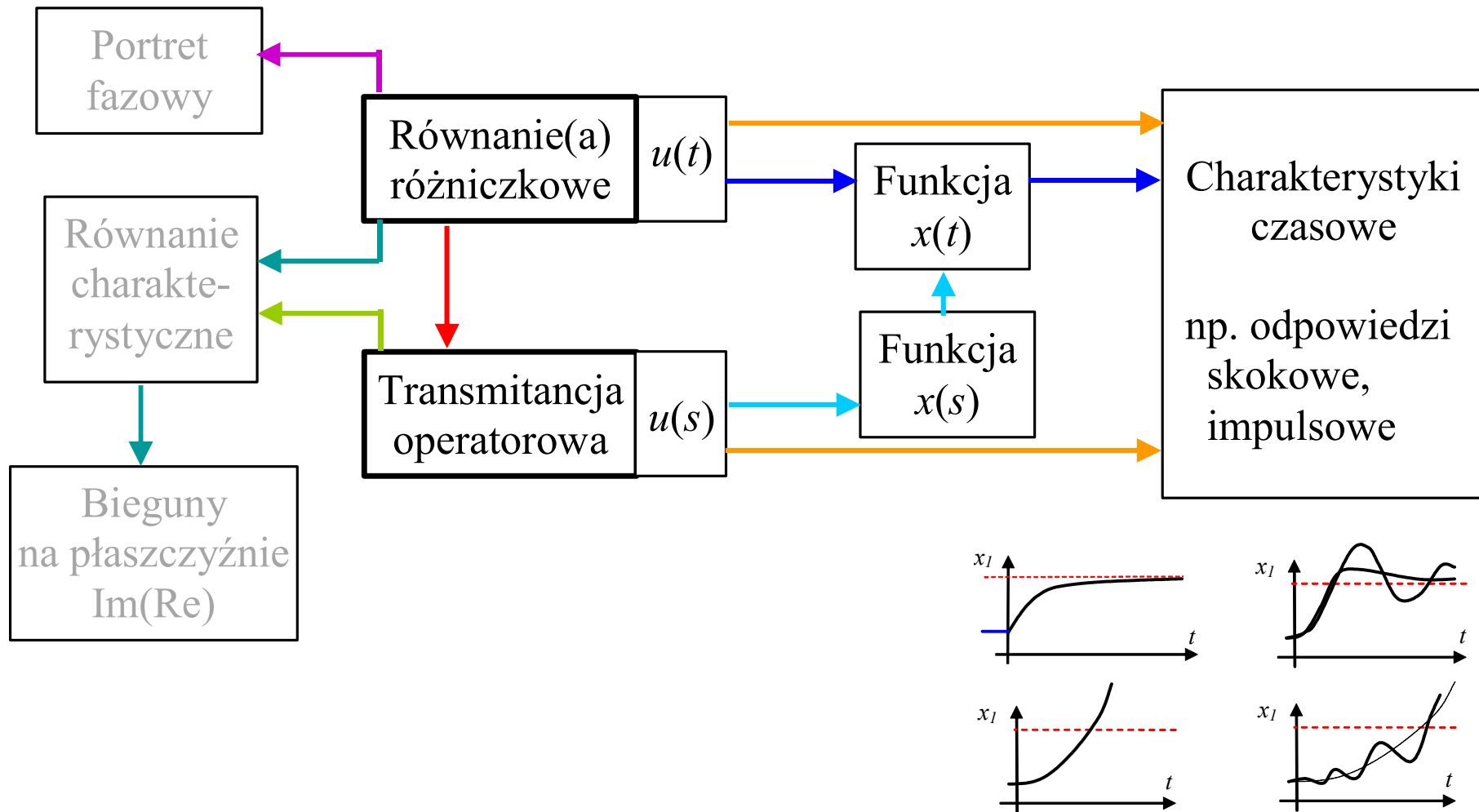


## Opis własności dynamicznych



Podstawowe człony (obiekty) dynamiki – najprostsze przypadki modeli dynamiki, przedstawiane zazwyczaj w postaci transmitancji

## Człon inercyjny

$$G(s) = \frac{k}{Ts + 1}, T > 0$$

$$a_1 \dot{x}(t) + a_0 x(t) = b_0 u(t)$$

$$G(s) = \frac{b_0}{a_1 s + a_0} = \frac{b_0}{a_0 \left( \frac{a_1}{a_0} s + 1 \right)}, k = \frac{b_0}{a_0}, T = \frac{a_1}{a_0}$$

r.s.:  $a_0 x = b_0 u \rightarrow x = b_0 u / a_0$

p.r. gdy  $u(t) = u_k$   $\lim_{s \rightarrow 0} s \frac{k}{Ts + 1} \frac{u_k}{s} = k u_k$

r.ch.:  $a_1 s + a_0 = 0 \rightarrow s_1 = \frac{-a_0}{a_1}$

$$Ts + 1 = 0 \rightarrow s_1 = \frac{-1}{T}$$

$$s_1 < 0 \rightarrow T > 0$$

Rozwiązanie ogólne dla  $u_k$ :

$$x(t) = A e^{(-1/T)t} + k u_k$$

a)  $x_s(t) = A e^{s_1 t} = A e^{(-1/T)t}$

b)  $x_w(t) = b_0 u_k / a_0 = k u_k$

Odpowiedź skokowa:

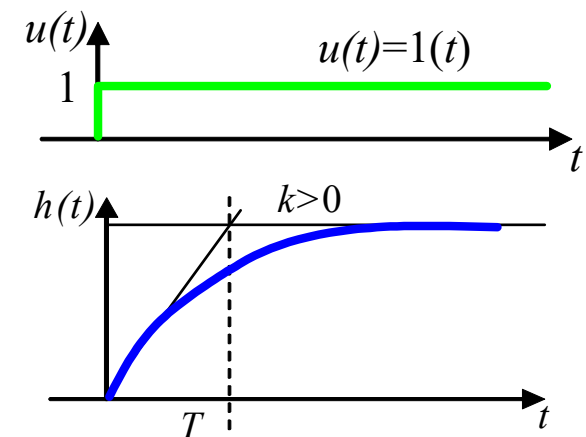
c)  $u_k = 1$

d)  $u(0) = 0, x(0) = 0$

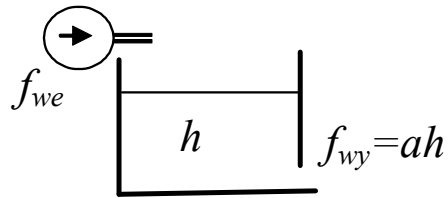
$$0 = A e^{-1/T \cdot 0} + k \rightarrow A = -k$$

$$h(t) = -k e^{(-1/T)t} + k$$

$$h(t) = k(1 - e^{(-1/T)t})$$



## Człon inercyjny - przykłady



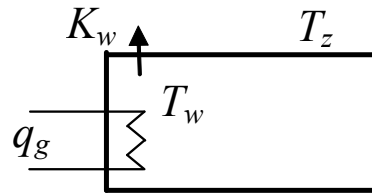
$$A\dot{h}(t) = f_{we}(t) - ah(t)$$

$$h(s) = \frac{1}{As + a} f_{we}(s)$$

$$k = \frac{1}{a}$$

$$T = \frac{A}{a} > 0$$

Gdy  $A \uparrow$ , to ...



$$C_V \dot{T}_w(t) = q_g(t) - K_{cw}(T_w(t) - T_z(t))$$

$$T_w(s) = \frac{1}{C_V s + K_{cw}} q_g(s) + \frac{K_{cw}}{C_V s + K_{cw}} T_z(s)$$

$$k = \frac{1}{K_{cw}}$$

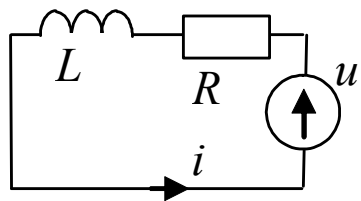
$$T = \frac{C_V}{K_{cw}} > 0$$

Gdy  $K_{cw} \uparrow$ , to ...

Gdy  $V \uparrow$ , to ...

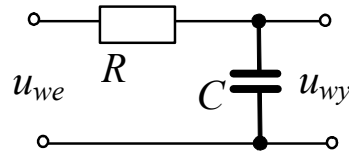
$$G(s) = \frac{k}{Ts + 1}, T > 0$$

Obiekty  
z samowyrównywaniem



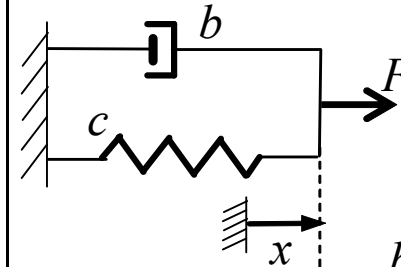
$$sLi(s) + Ri(s) = u(s)$$

$$i(s) = \frac{1}{sL + R} u(s)$$



$$u_{wy}(s) = \frac{1}{sC} \frac{u_{we}(s)}{R + 1/(sC)}$$

$$u_{wy}(s) = \left( \frac{1}{sCR + 1} \right) u_{we}(s)$$



$$b\dot{x}(t) + cx(t) = F(t)$$

$$x(s) = \frac{1}{sb + c} F(s)$$

## Człon całkujący

$$a_1 \dot{x}(t) = b_0 u(t)$$

$$G(s) = \frac{b_0}{a_1 s}, \quad T_i = \frac{a_1}{b_0}$$

$$G(s) = \frac{1}{T_i s} = k_i \frac{1}{s}$$

r.s.:

p.r. gdy  $u(t) = u_k \quad \lim_{s \rightarrow 0} s \frac{1}{T_i s} \frac{u_k}{s} = \infty$

r.ch.:  $a_1 s = 0 \rightarrow s = 0$

p.r. gdy  $u(t) = \delta(t) \quad \lim_{s \rightarrow 0} s \frac{1}{T_i s} 1 = \frac{1}{T_i}$

Rozwiązanie ogólne dla  $u_k$ :

$$x(t) = k_i u_k t + x_0$$

a)  $x(t) = k_i \int u_k dt + x_0$

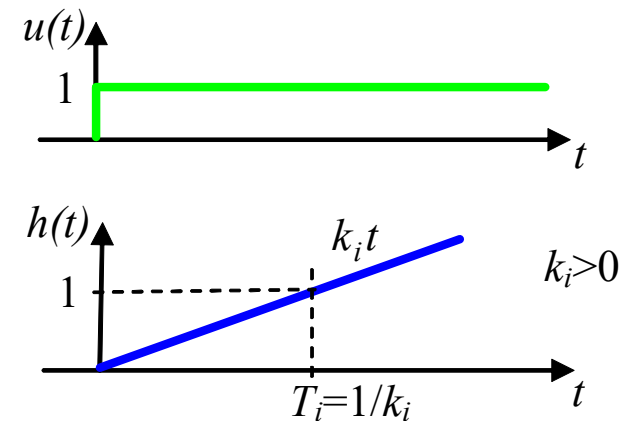
Odpowiedź skokowa:

$$h(t) = k_i t$$

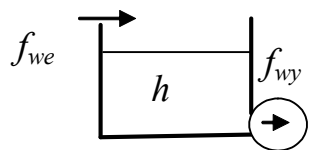
b)  $u_k = 1$

c)  $u(0) = 0, x(0) = 0$

$$0 = k_i t + x_k$$

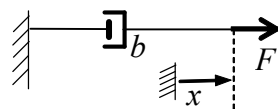


## Człon całkujący - przykłady



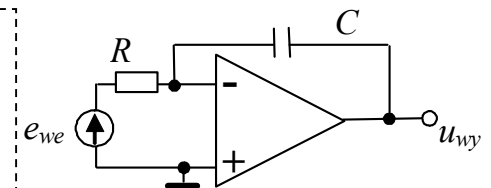
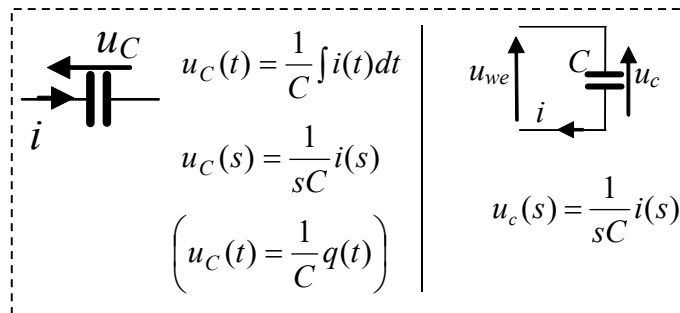
$$A \dot{h}(t) = f_{we}(t) - f_{wy}(t)$$

$$h_1(s) = \frac{1}{As} f_{we}(s) - \frac{1}{As} f_{wy}(s)$$



$$b \dot{x}(t) = F(t)$$

$$x(s) = \frac{1}{sb} F(s)$$



$$u_{wy} = \frac{-1}{sRC} e_{we} \quad 4$$

## Człon oscylacyjny

$$a_2 \ddot{x}(t) + a_1 \dot{x}(t) + a_0 x(t) = b_0 u(t)$$

$$G(s) = \frac{b_0}{a_2 s^2 + a_1 s + a_0} = \frac{b_0}{a_2 \left( s^2 + \frac{a_1}{a_2} s + \frac{a_0}{a_2} \right)}$$

$$G(s) = \frac{k_1}{s^2 + 2 \xi \omega_n s + \omega_n^2}, \omega_n > 0$$

$$G(s) = \frac{k}{T_n^2 s^2 + 2 \xi T_n s + 1}, T_n > 0$$

$$= \frac{b_0}{a_0 \left( \frac{a_2}{a_0} s^2 + \frac{a_1}{a_0} s + 1 \right)}$$

Jeśli  $a_0/a_2 > 0$ , to:  $\omega_n^2 = \frac{a_0}{a_2}$ ,  $2\xi\omega_n = \frac{a_1}{a_2}$ ,  $k_1 = \frac{b_0}{a_2}$

$$T^2 = \frac{a_2}{a_0}, \quad 2\xi T = \frac{a_1}{a_0}, \quad k = \frac{b_0}{a_0}$$

r.s.:  $a_0 x = b_0 u$

$$\lim_{s \rightarrow 0} s \frac{k_1}{s^2 + 2\xi\omega_n s + \omega_n^2} \frac{u_0}{s} = \frac{k_1 u_0}{\omega_n^2}, \quad u(t) = u_0$$

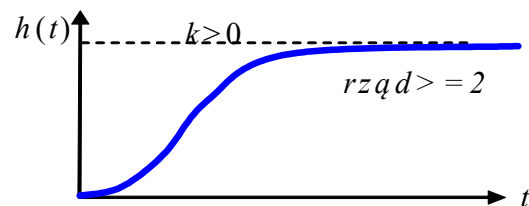
r.ch.:  $a_2 s^2 + a_1 s + a_0 = 0 \rightarrow s_1, s_2$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad T^2 s^2 + 2\xi T s + 1 = 0$$

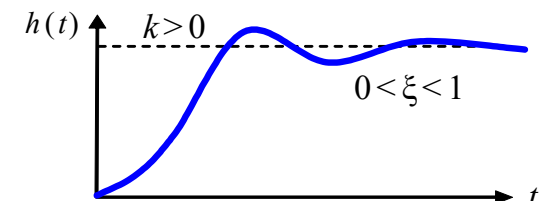
$\text{Im}(s_i) = 0, \text{Re}(s_i) < 0$

$$\frac{k}{(T_1 s + 1)(T_2 s + 1)} \longleftarrow G(s) = \frac{b_0}{a_2 (s - s_1)(s - s_2)}$$

$$\frac{k_1}{s^2 + 2 \xi \omega s + \omega^2}$$



Człon inercyjny  
II rzędu



## Człon oscylacyjny znormalizowany



$$\frac{\omega_n^2}{s^2 + 2 \xi \omega_n s + \omega_n^2}, \omega_n > 0$$

$$\frac{1}{T_n^2 s^2 + 2 \xi T_n s + 1}, T_n > 0$$

$$\frac{\sigma^2 + \omega_r^2}{s^2 + 2 \sigma s + \sigma^2 + \omega_r^2}$$

$$s_1 = -\xi \omega_n + \omega_n \sqrt{\xi^2 - 1}$$

$$s_2 = -\xi \omega_n - \omega_n \sqrt{\xi^2 - 1}$$

Gdy  
 $\xi^2 - 1 < 0$

$$s_1 = \alpha + j \omega_r$$

$$s_2 = \alpha - j \omega_r$$

gdzie:

$$\alpha = -\xi \omega_n$$

$$\omega_r = \omega_n \sqrt{1 - \xi^2}$$

$$s_1 = -\sigma + j \omega_r$$

$$s_2 = -\sigma - j \omega_r$$

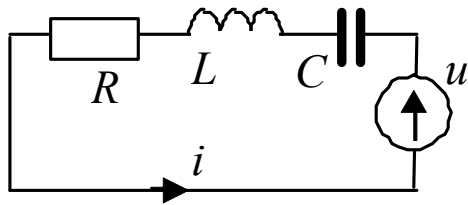
gdzie:

$$\sigma = \xi \omega_n$$

$$\omega_r = \omega_n \sqrt{1 - \xi^2}$$

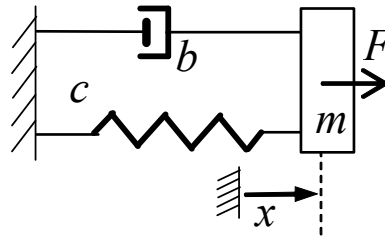
$$\frac{\sigma^2 + \omega_r^2}{s^2 + 2 \sigma s + \sigma^2 + \omega_r^2} = \frac{(\xi \omega_n)^2 + (\omega_n \sqrt{1 - \xi^2})^2}{s^2 + 2 \xi \omega_n s + (\xi \omega_n)^2 + (\omega_n \sqrt{1 - \xi^2})^2} = \frac{\omega_n^2}{s^2 + 2 \xi \omega_n s + \omega_n^2}$$

## Człon oscylacyjny - przykłady



$$sLi(s) + Ri(s) + \frac{1}{sC}i(s) = u(s)$$

$$\frac{i(s)}{u(s)} = \frac{sC}{s^2LC + sRC + 1}$$

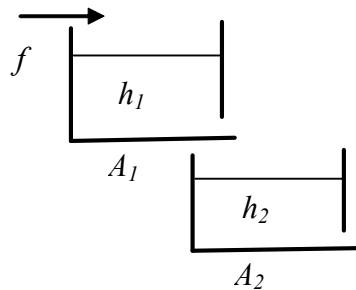


$$m\ddot{x}(t) + b\dot{x}(t) + cx(t) = F(t)$$

$$\frac{x(s)}{F(s)} = \frac{1}{ms^2 + bs + c}$$

$$G(s) = \frac{k_1}{s^2 + 2\xi\omega_n s + \omega_n^2}, \omega_n > 0$$

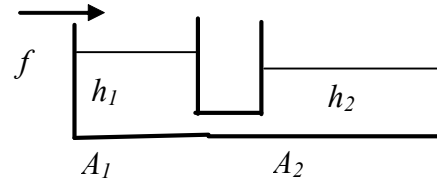
$$G(s) = \frac{k}{T_n^2 s^2 + 2\xi T_n s + 1}, T_n > 0$$



$$\begin{cases} A_1 \dot{h}_1(t) = f(t) - a_1 h_1(t) \\ A_2 \dot{h}_2(t) = a_1 h_1(t) - a_2 h_2(t) \end{cases}$$

$$\frac{h_2(s)}{f(s)} = \frac{1}{(A_1 s + a_1)(A_2 s + a_2)}$$

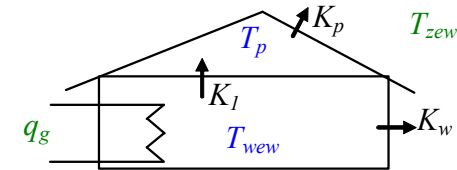
$$\xi > 1$$



$$\begin{cases} A_1 \dot{h}_1(t) = f(t) - a_1(h_1(t) - h_2(t)) \\ A_2 \dot{h}_2(t) = a_1(h_1(t) - h_2(t)) - a_2 h_2(t) \end{cases}$$

$$\frac{h_2(s)}{f(s)} = \frac{a_1}{(A_1 s + a_1)(A_2 s + a_1 + a_2) - a_1^2}$$

$$\xi = ?$$



$$\xi = ?$$

## Człon proporcjonalny

$$a_0 x(t) = b_0 u(t)$$

$$G(s) = k, \quad k = \frac{b_0}{a_0}$$

$$G(s) = k$$

r.s.:  $a_0 x = b_0 u$

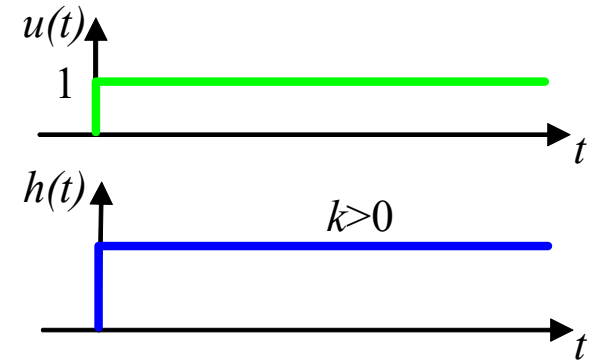
r.ch.:

Odpowiedź skokowa:

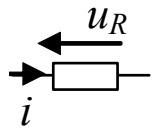
$$x(t) = k1(t)$$

a)  $u_k = 1$

b)  $u(0)=0, x(0)=0$

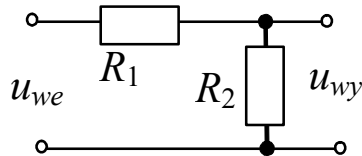


## Człon proporcjonalny - przykłady



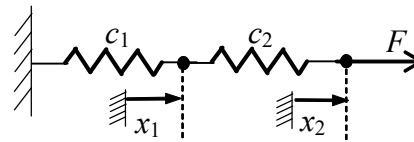
$$u_R(t) = Ri(t)$$

$$\frac{u_R(s)}{i(s)} = R$$



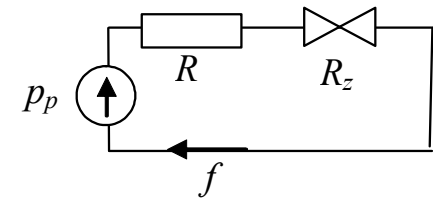
$$u_{wy}(t) = \left( \frac{R_2}{R_1 + R_2} \right) u_{we}(t)$$

$$\frac{u_{wy}(s)}{u_{we}(s)} = \frac{R_2}{R_1 + R_2}$$



$$\begin{cases} 0 = c_1 x_1(t) + c_2 (x_1(t) - x_2(t)) \\ F(t) = c_2 (x_2(t) - x_1(t)) \end{cases}$$

$$x_1(t) = \frac{1}{c_1} F(t) \quad x_2(t) = \frac{c_1 + c_2}{c_1 c_2} F(t)$$



$$p_p(t) = (R + R_z) f(t)$$

$$f(t) = \frac{1}{R + R_z} p_p(t)$$



## Człon różniczkujący

$$a_0 x(t) = b_1 \dot{u}(t)$$

$$G(s) = T_d s, \quad T_d = \frac{b_1}{a_0}$$

$$G(s) = T_d s$$

r.s.:  $a_0 x = 0$

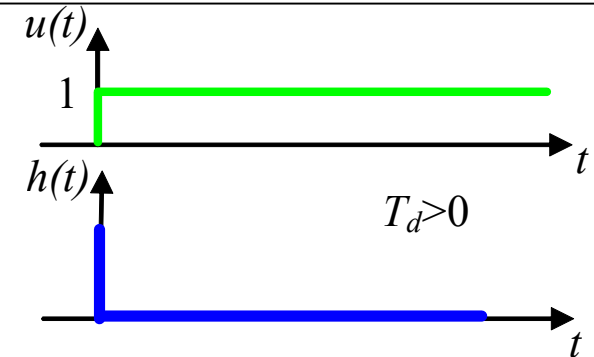
r.ch.:

Odpowiedź skokowa:

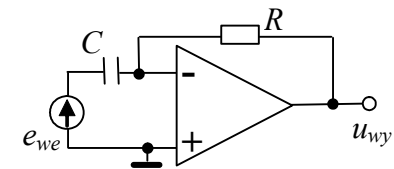
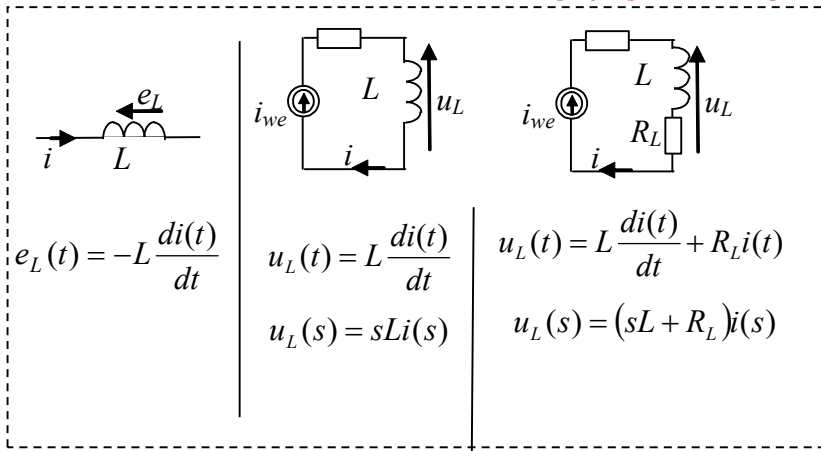
$$x(t) = T_d \delta(t)$$

a)  $u_k = 1$

b)  $u(0) = 0, x(0) = 0$



## Człon różniczkujący - przykłady



$$u_{wy} = -sRC e_{we}$$

## Podstawowe obiekty (człony) dynamiki

$$a_0 x(t) = b_0 u(t)$$

$$G(s) = k$$

$$a_1 \dot{x}(t) + a_0 x(t) = b_0 u(t)$$

$$G(s) = \frac{k}{Ts + 1}, T > 0$$

$$\ddot{x}(t) + 2\xi\omega_n \dot{x}(t) + \omega_n^2 x(t) = b_0 u(t)$$

$$G(s) = \frac{k_1}{s^2 + 2\xi\omega_n s + \omega_n^2}, \omega_n > 0$$

$$G(s) = \frac{k}{T_n^2 s^2 + 2\xi T_n s + 1}, T_n > 0$$

$$a_1 \dot{x}(t) = b_0 u(t)$$

$$G(s) = \frac{1}{T_i s}$$

$$a_0 x(t) = b_1 \dot{u}(t)$$

$$G(s) = T_d s$$

$k, k_1$  – współczynniki wzmocnienia członu

$T$  – stała czasowa

$T_o$  – opóźnienie

$T_i$  – czas całkowania

$T_d$  – czas różniczkowania

$\omega_n$  – pulsacja drgań własnych nietłumionych

$T_n$  – okres drgań własnych nietłumionych  
(współczynnik okresu drgań własnych)

$$\omega_n = \frac{1}{T_n}$$

$$\omega_n = 2\pi f = \frac{2\pi}{T}$$

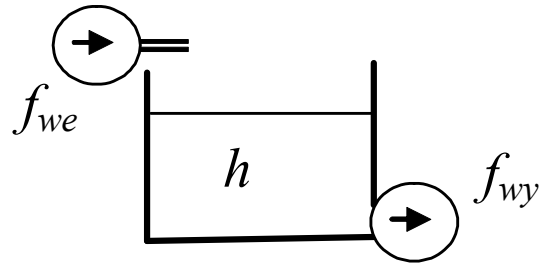
$\xi$  – współczynnik tłumienia względnego

$$G(s) = \frac{L(s)}{M(s)}$$

$$x(t) = u(t - T_0)$$

$$G(s) = e^{-T_0 s}$$

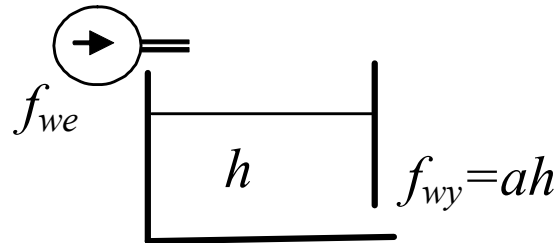
## Przykłady obiektów bez/z samowyrównywaniem



$$A\dot{h}(t) = f_{we}(t) - f_{wy}(t)$$

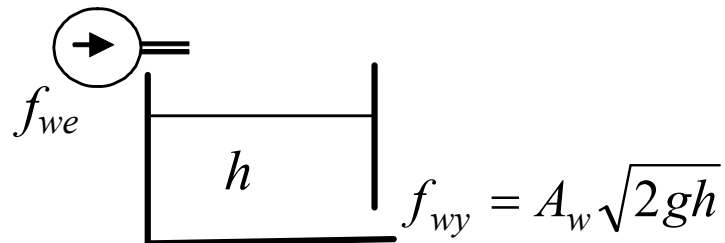
$$h_1(s) = \frac{1}{As} f_{we}(s) - \frac{1}{As} f_{wy}(s)$$

samowyrównywanie

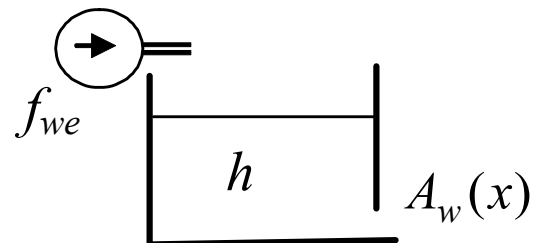


$$A\dot{h}(t) = f_{we}(t) - ah(t)$$

$$h(s) = \frac{1}{As + a} f_{we}(s)$$

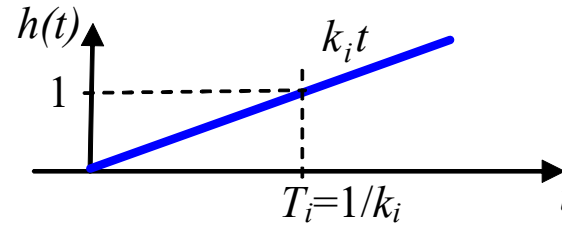
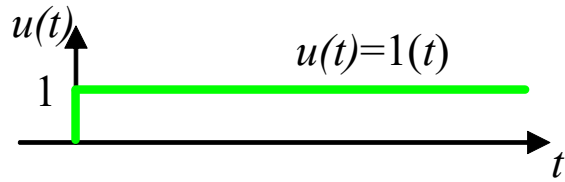


$$A\dot{h}(t) = f_{we}(t) - A_w \sqrt{2gh(t)}$$

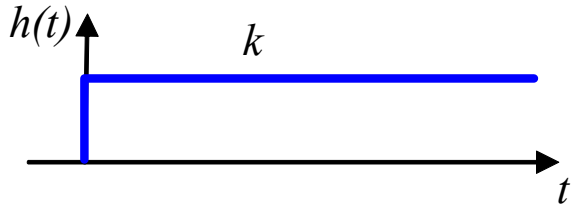


$$A\dot{h}(t) = f_{we}(t) - A_w(x(t)) \sqrt{2gh(t)}$$

## Odpowiedzi skokowe członów podstawowych



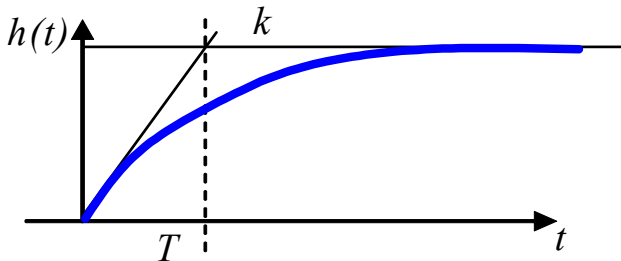
$$\frac{1}{T_i s} = \frac{k_i}{s}$$



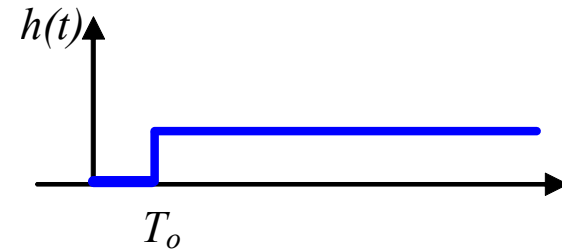
$k$



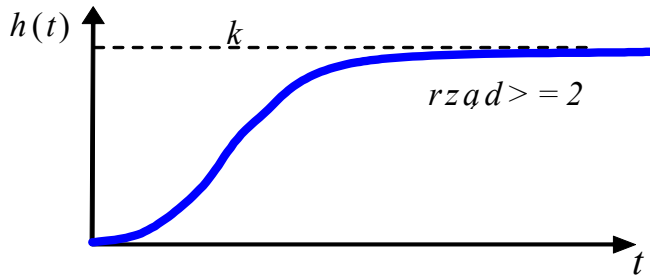
$T_d s$



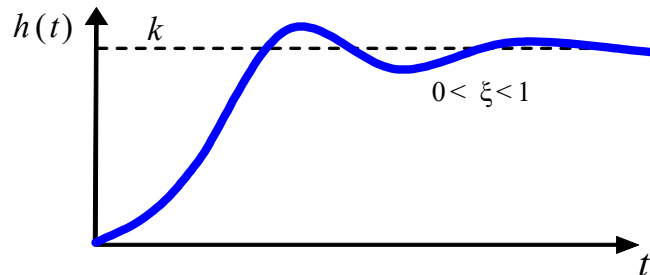
$$\frac{k}{Ts + 1}$$



$e^{-T_o s}$



$$\frac{k}{(T_1 s + 1)(T_2 s + 1)}$$

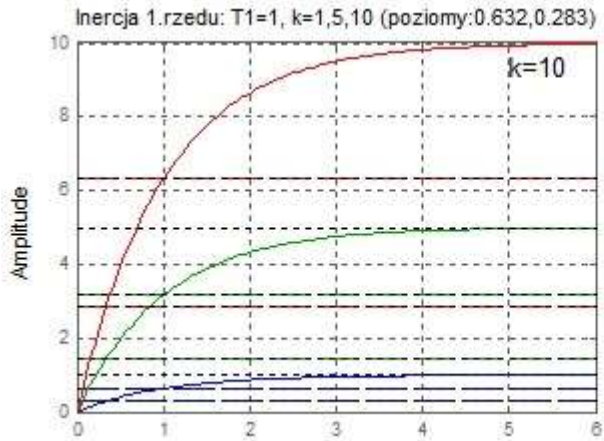


$$\frac{k}{s^2 + 2 \xi \omega_n s + \omega_n^2}$$

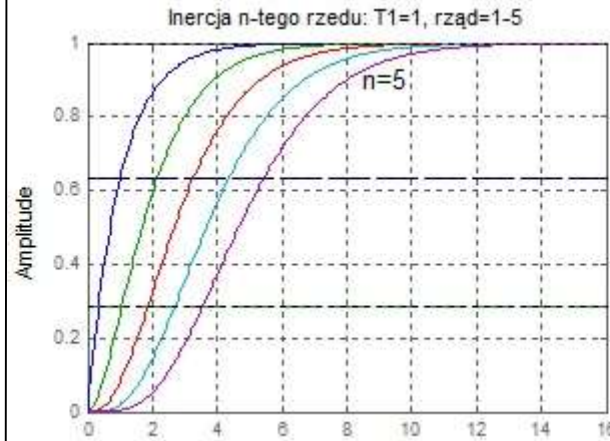
$$\frac{K}{T_n^2 s^2 + 2 \xi T_n s + 1}$$

# Odpowiedzi skokowe – wpływ parametrów

$$\frac{k}{(T_1s + 1)}$$

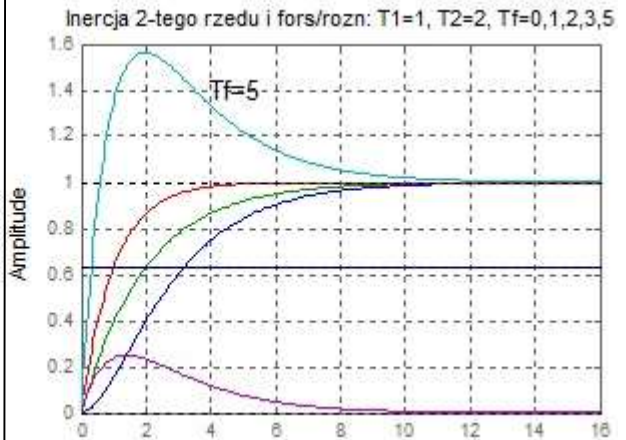
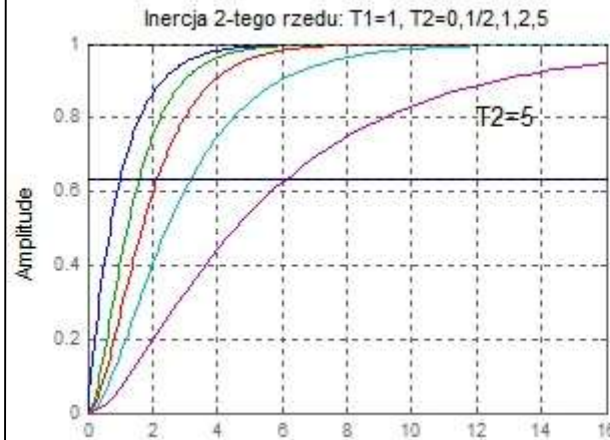
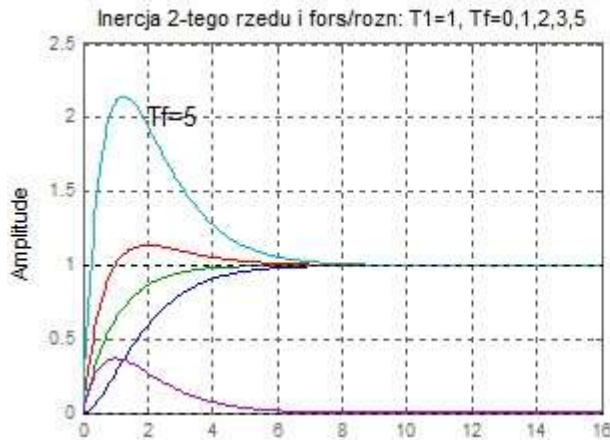
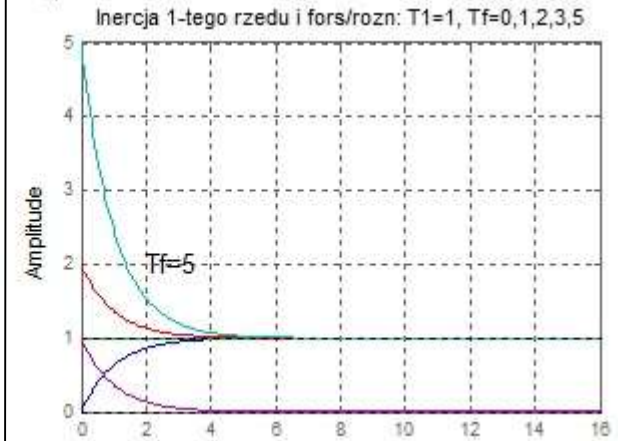


$$\frac{k}{(T_1s + 1)^n}$$



$$\frac{k(T_f s + 1)}{(T_1 s + 1)}$$

$$\frac{ks}{(T_1 s + 1)}$$



$$\frac{k(T_f s + 1)}{(T_1 s + 1)^2}$$

$$\frac{ks}{(T_1 s + 1)^2}$$

$$\frac{k}{(T_1 s + 1)(T_2 s + 1)}$$

$$\frac{k(T_f s + 1)}{(T_1 s + 1)(T_2 s + 1)}$$

$$\frac{ks}{(T_1 s + 1)(T_2 s + 1)}$$