Metoda klasyczna $b\dot{x}(t) + cx(t) = u(t)$

$$u(t) = \sin \omega t$$

1) równanie jednorodne: $b\dot{x}_s(t) + cx_s(t) = 0$

4) równanie charakterystyczne: $b\lambda + c = 0$

5) pierwiastki równania charakt.: $\lambda_1 = -c/b$

rozwiązanie swobodne: $x_s(t) = A_1 e^{-(c/b)t}$

---

1) równanie niejednorodne: $b\dot{x}_w(t) + cx_w(t) = \sin \omega t$

2) wymuszenie i pochodne: $sin\ \omega t,\ \omega\ cos\ \omega t$

3) postać $x_w(t)$:

$$x_w(t) = C_1 \sin \omega t + C_2 \cos \omega t$$

$$\dot{x}_w(t) = \omega C_1 \cos \omega t - \omega C_2 \sin \omega t$$

4) podstawienie $x_w(t)$:

$$b(\omega C_1 \cos \omega t - \omega C_2 \sin \omega t) + c(C_1 \sin \omega t + C_2 \cos \omega t) = \sin \omega t$$

5) uporządkowanie 6) układ równań 7) wyznaczenie stałych

rozwiązanie wymuszone: $x_w(t) = \dfrac{c}{b^2\omega^2 + c^2} \sin \omega t - \dfrac{b\omega}{b^2\omega^2 + c^2} \cos \omega t$

$$x_w(t) = 1/\sqrt{b^2\omega^2 + c^2}\, \sin\left(\omega t - arctg\, \frac{b\omega}{c}\right)$$

Metoda klasyczna $b\dot{x}(t) + cx(t) = u(t)$

$$u(t) = \sin\omega t$$

$$x(t) = A_1 e^{-(c/b)t} + \frac{c}{b^2\omega^2 + c^2}\sin\omega t - \frac{b\omega}{b^2\omega^2 + c^2}\cos\omega t$$

$$x(t) = A_1 e^{-(c/b)t} + 1/\sqrt{b^2\omega^2 + c^2}\,\sin\left(\omega t - arctg\frac{b\omega}{c}\right)$$

$x(0)=0$

$$0 = A_1 e^{-(c/b)0} + \frac{c}{b^2\omega^2 + c^2}\sin\omega 0 - \frac{b\omega}{b^2\omega^2 + c^2}\cos\omega 0$$

$$A_1 = \frac{b\omega}{b^2\omega^2 + c^2}$$

$$x(t) = \frac{b\omega}{b^2\omega^2 + c^2}e^{-(c/b)t} + \frac{c}{b^2\omega^2 + c^2}\sin\omega t - \frac{b\omega}{b^2\omega^2 + c^2}\cos\omega t$$

$$x(t) = \frac{b\omega}{b^2\omega^2 + c^2}e^{-(c/b)t} + 1/\sqrt{b^2\omega^2 + c^2}\,\sin\left(\omega t - \mathrm{arctg}\frac{b\omega}{c}\right)$$

Transmitancja widmowa

G(s) = G(jω),

$$G(j\omega)=\frac{P_1(\omega)+jQ_1(\omega)}{P_2(\omega)+jQ_2(\omega)}=\frac{[P_1(\omega)+jQ_1(\omega)][P_2(\omega)-jQ_2(\omega)]}{P_2^2(\omega)+Q_2^2(\omega)}=P(\omega)+jQ(\omega)$$

Postać algebraiczna

$$G(j\omega)=P(\omega)+jQ(\omega)$$

$$P(\omega)=M(\omega)\cos[\varphi(\omega)]$$

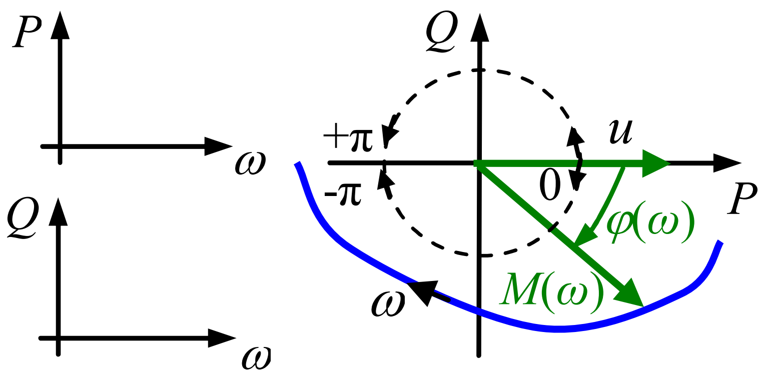
$$Q(\omega)=M(\omega)\sin[\varphi(\omega)]$$

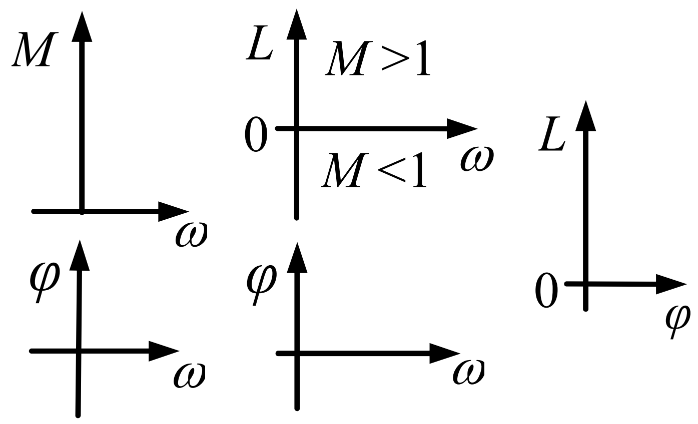
Postać wykładnicza

$$G(j\omega)=M(\omega)e^{j\varphi(\omega)}$$

$$M(\omega)=|G(j\omega)|=\sqrt{P^2(\omega)+Q^2(\omega)}$$

$$\varphi(\omega)=\arg[G(j\omega)]=arctg\frac{Q(\omega)}{P(\omega)}$$

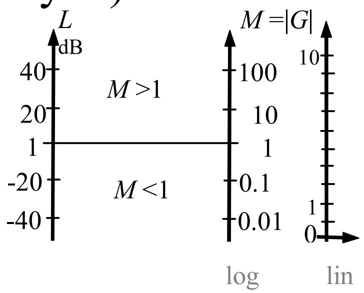




Charakterystyki częstotliwościowe

$$G(j\omega) = P(\omega) + jQ(\omega) = M(\omega)e^{j\varphi(\omega)}$$

ch-ka rzeczywista	- $P(\omega) = \mathrm{Re}(G(j\omega))$
ch-ka urojona	- $Q(\omega) = \mathrm{Im}(G(j\omega))$
ch. aplitudowo-fazowa	- $Q(P)$ (ch.Nyquista - dla ukł. otwartych)
ch. amplitudowa	- $M(\omega) = \lvert G(j\omega)\rvert$
ch. fazowa	- $\varphi(\omega)$
logarytmiczna ch. modułu	- $L(\omega) = 20\ \lg M(\omega) = 20\ \lg\lvert G(j\omega)\rvert$
logarytmiczna ch. fazy	- $\varphi(\omega) = \mathrm{arctg}(Q/P)$
log.ch.amplitudowo-fazowa	- $L(\varphi)$



Charakterystyki członów połączonych szeregowo

$$G(j\omega) = \prod_{i=1}^{n} G_i(j\omega) = \prod \left[M_i(\omega)e^{j\varphi_i(\omega)}\right]$$

- ch. amplitudowa $$M(\omega) = \prod M_i(\omega)$$

- ch. fazowa $$\varphi(\omega) = \sum \varphi_i(\omega)$$

- logarytmiczna ch.aplitudowa $$L(\omega) = 20\lg\left(\prod M_i(\omega)\right) = \sum L_i(\omega)$$

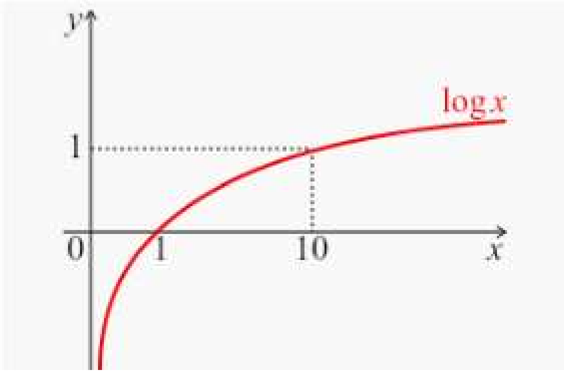
- logarytmiczna ch. fazy $$\varphi(\omega) = \sum \varphi_i(\omega)$$

Logarytmy

Definicja:

$$\log_a b = c \Leftrightarrow a^c = b,$$

$$a, b > 0, a \neq 1$$



Wybrane własności:

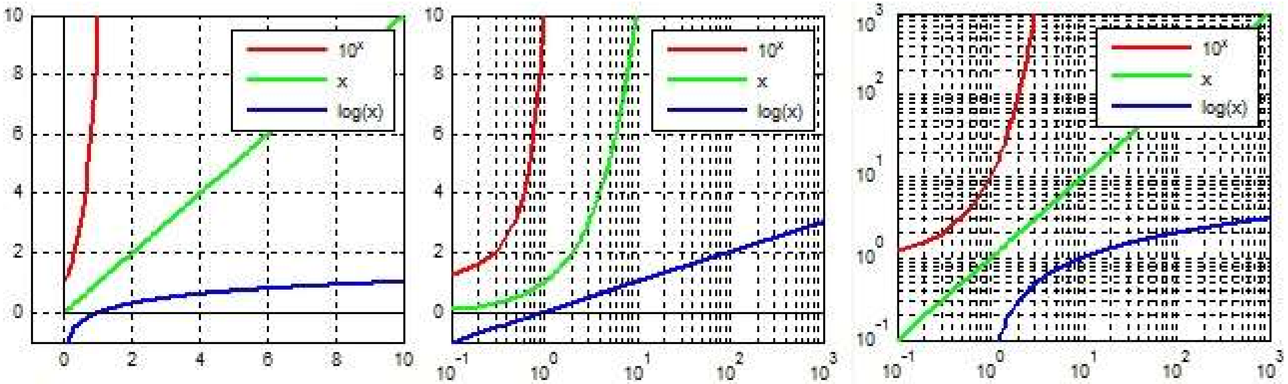
$$\log_a(bc) = \log_a b + \log_a c$$

$$\log_a \frac{1}{b} = -\log_a b$$

$$\log_a \frac{b}{c} = \log_a b - \log_a c$$

$$\log_a b^n = n \log_a b$$

Zastosowanie na skali logarytmicznej (na przykładzie: $y_1(x)=10^x$, $y_2(x)=x$, $y_3(x)=\lg(x)$)



$$P_B = \log_{10}\left(\frac{P}{P_0}\right)$$

bel (zwiększenie mocy P względem mocy P_0)

$$1\,\mathrm{dB} = \frac{1}{10}\,\mathrm{B}$$

decybel

$$P_{\mathrm{dB}} = 10\log_{10}\left(\frac{P}{P_0}\right)$$

$$L\,[\mathrm{dB}] = 10\log_{10}\left(\frac{A^2}{A_0^2}\right) = 20\log_{10}\left(\frac{A}{A_0}\right)$$

Amplituda sygnału jest proporcjonalna do kwadratu mocy

Przykład: Moc sygnały wyjściowego jest 10^6 większa od sygnału wejściowego:

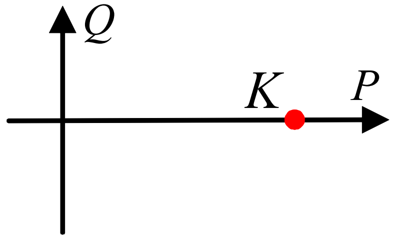
$$\log_{10}\left(\frac{1.000.000 \cdot P_{prg}}{P_{prg}}\right) = \log\left(10^6 \cdot \frac{P_{prg}}{P_{prg}}\right) = \log_{10}\left(10^6\right) = 6\,\mathrm{B} = 60\,\mathrm{dB}$$

cz. proporcjonalny:

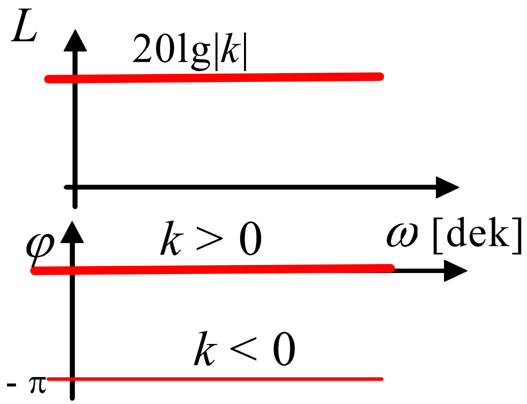
$$P(\omega) + jQ(\omega) = M(\omega)e^{j\varphi(\omega)}$$
$$L(\omega) = 20\lg M(\omega)$$
$$\varphi(\omega) = arctg(Q/P)$$

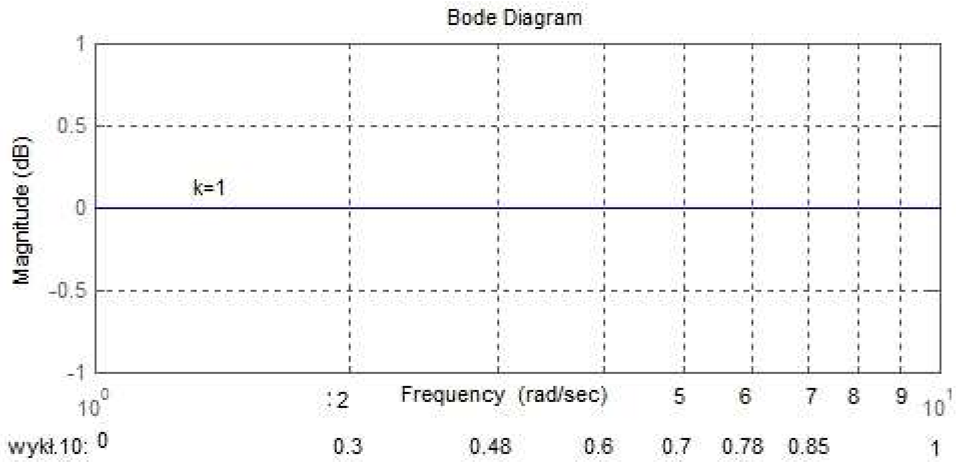
$$G(s) = K \qquad\qquad G(j\omega) = K$$

$$P(\omega) = K$$
$$Q(\omega) = 0$$



$$M(\omega) = |K|$$
$$\varphi(\omega) = 0$$
$$L(\omega) = 20\lg|K|$$



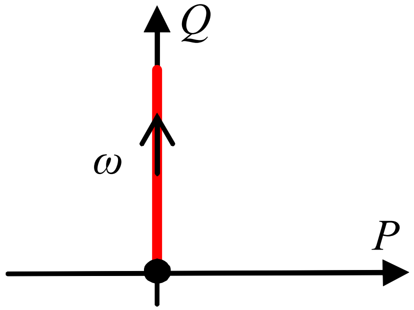


cz. różniczkowy:

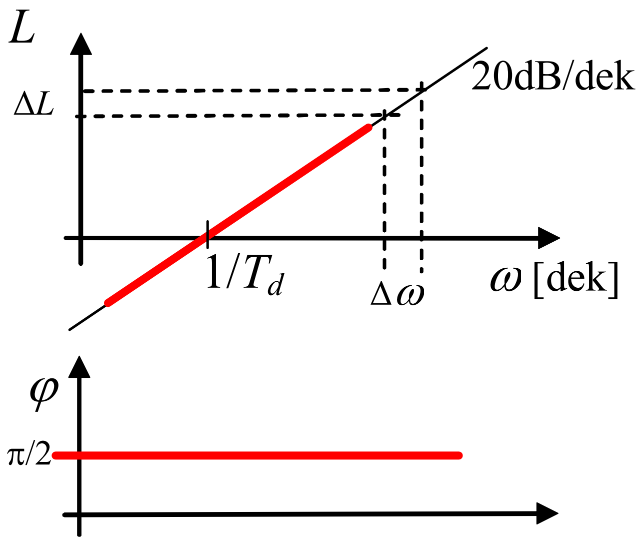
$$G(s) = sT_d \qquad G(j\omega) = j\omega T_d$$

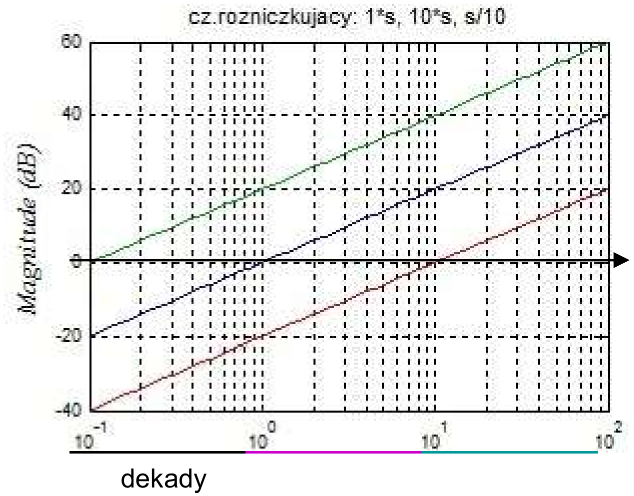
$$P(\omega) + jQ(\omega) = M(\omega)e^{j\varphi(\omega)}$$
$$L(\omega) = 20\lg M(\omega)$$
$$\varphi(\omega) = arctg(Q/P)$$

$$P(\omega) = 0$$
$$Q(\omega) = \omega T_d$$



$$M(\omega) = |j\omega T_d| = \omega T_d$$
$$\varphi(\omega) = \pi/2$$
$$L(\omega) = 20\lg|\omega T_d|$$





Legenda ???

$$L = 0 \rightarrow M = 1 \rightarrow \omega T_d = 1 \rightarrow \omega = \frac{1}{T_d}$$

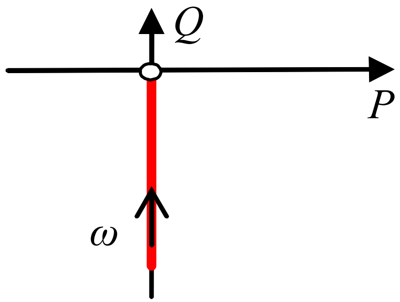
$$\frac{\Delta L}{\Delta\omega} = 20\lg(10\omega_1) - 20\lg(\omega_1) = 20\lg\frac{10\omega_1}{\omega_1} = 20\frac{dB}{dek}$$

cz. całkujący:

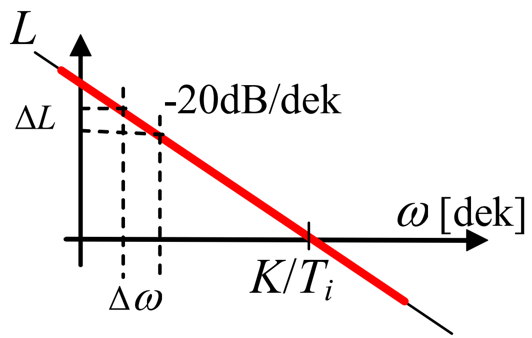
$$P(\omega)+jQ(\omega)=M(\omega)e^{j\varphi(\omega)}$$
$$L(\omega)=20\lg M(\omega)$$
$$\varphi(\omega)=arctg\left(Q/P\right)$$

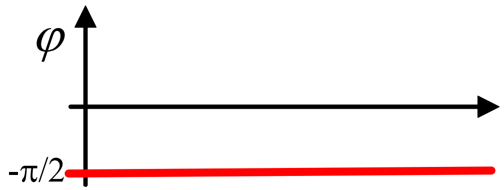
$$G(s)=\frac{K}{sT_i} \qquad G(j\omega)=\frac{K}{j\omega T_i}=-j\frac{K}{\omega T_i}$$

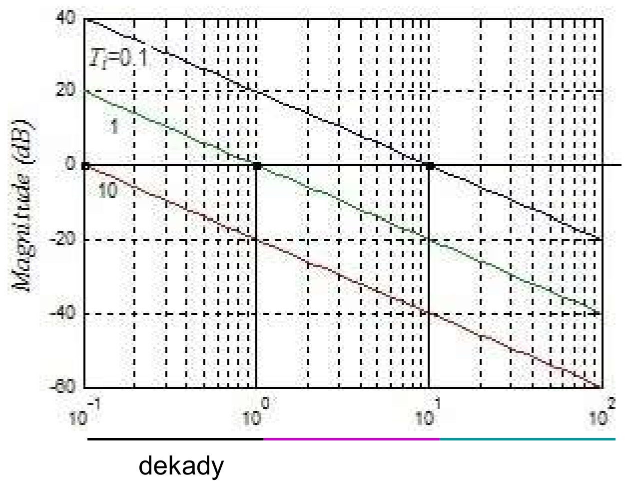
$$P(\omega)=0$$
$$Q(\omega)=-\frac{K}{\omega T_d}$$



$$A(\omega)=\left|\frac{K}{j\omega T_i}\right|=\frac{K}{\omega T_i}$$
$$\varphi(\omega)=-\pi/2$$
$$L(\omega)=20\lg\left|\frac{K}{\omega T_i}\right|=$$
$$=20\lg K-20\lg\omega-20\lg T_i$$







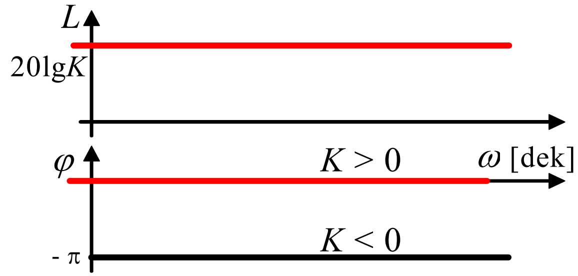
$$L=0\rightarrow M=1\rightarrow\frac{K}{\omega T_i}=1\rightarrow\omega=\frac{K}{T_i}$$

$$\frac{\Delta L}{\Delta\omega}=-20\lg(10\omega_1)+20\lg(\omega_1)=20\lg\frac{\omega_1}{10\omega_1}=-20\frac{dB}{dek}$$

cz. proporcjonalny:

$$G(s) = K \qquad G(j\omega) = K$$

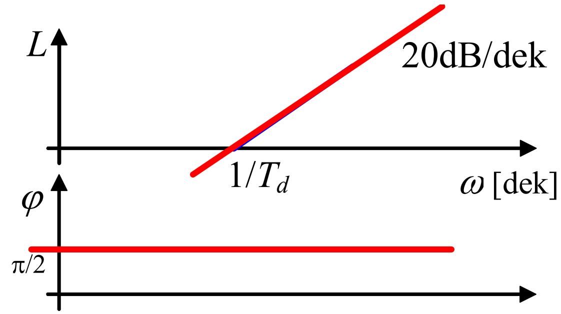
$$L(\omega) = 20\lg|K|$$



cz. różniczkowy:

$$G(s) = sT_d \qquad G(j\omega) = j\omega T_d$$

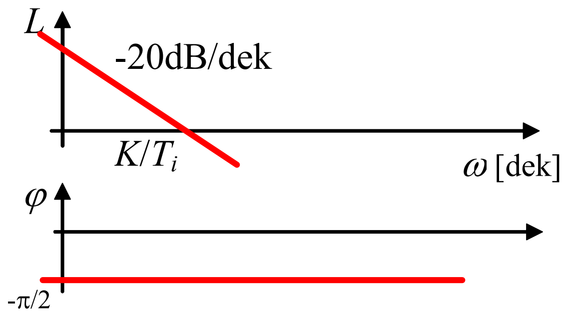
$$L(\omega) = 20\lg|\omega T_d|$$



cz. całkujący:

$$G(s) = \frac{K}{sT_i} \qquad G(j\omega) = \frac{K}{j\omega T_i} = -j\frac{K}{\omega T_i}$$

$$L(\omega) = 20\lg\left|\frac{K}{\omega T_i}\right|$$

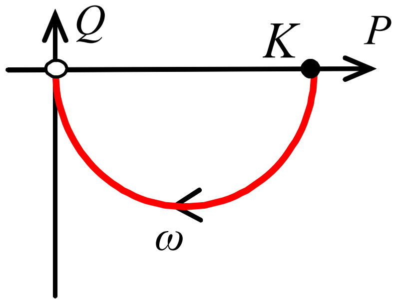


cz. inercyjny:

$$P(\omega)+jQ(\omega)=M(\omega)e^{j\varphi(\omega)}$$
$$L(\omega)=20\lg M(\omega)$$
$$\varphi(\omega)=arctg(Q/P)$$

$$G(s)=\frac{K}{1+sT} \qquad G(j\omega)=\frac{K}{1+j\omega T}=\frac{K}{1+\omega^2T^2}-j\frac{K\omega T}{1+\omega^2T^2}$$

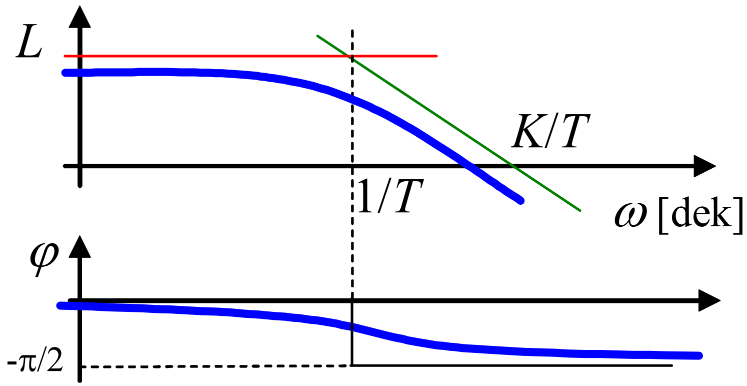
$$P(\omega)=\frac{K}{1+\omega^2T^2}$$
$$Q(\omega)=-\frac{K\omega T}{1+\omega^2T^2}$$



$$M(\omega)=\left|\frac{K}{1+j\omega T}\right|$$
$$\varphi(\omega)=arctg(-\omega T)$$

dla ω<<1/T $\quad G(j\omega)\approx K$

dla ω>>1/T $\quad G(j\omega)\approx\frac{K}{j\omega T}$

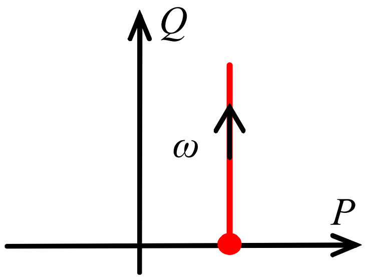


cz. forsujący:

$$P(\omega) + jQ(\omega) = M(\omega)e^{j\varphi(\omega)}$$
$$L(\omega) = 20\lg M(\omega)$$
$$\varphi(\omega) = arctg(Q/P)$$

$$G(s) = 1 + sT \qquad G(j\omega) = 1 + j\omega T$$

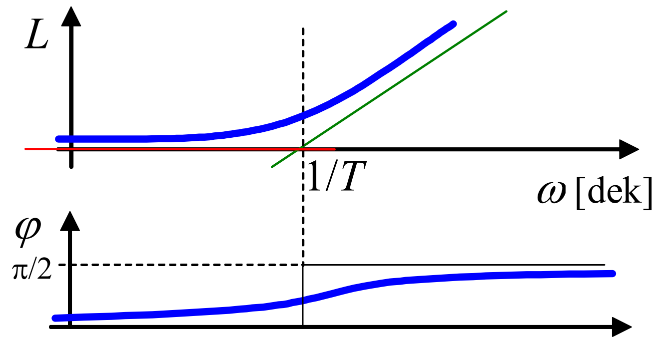
$$P(\omega) = 1$$
$$Q(\omega) = \omega T$$



$$M(\omega) = |1 + j\omega T|$$
$$\varphi(\omega) = arctg(\omega T)$$

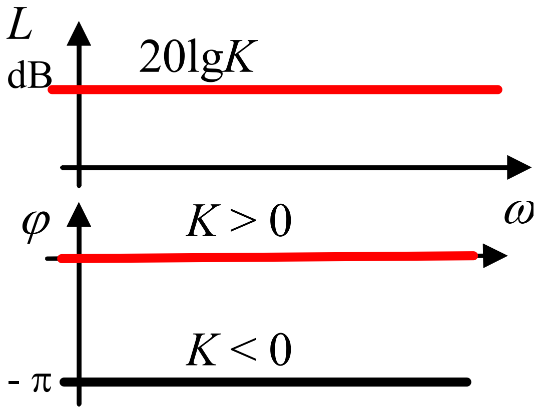
dla ω<<1/T $\quad G(j\omega) \approx 1$

dla ω>>1/T $\quad G(j\omega) \approx j\omega T$

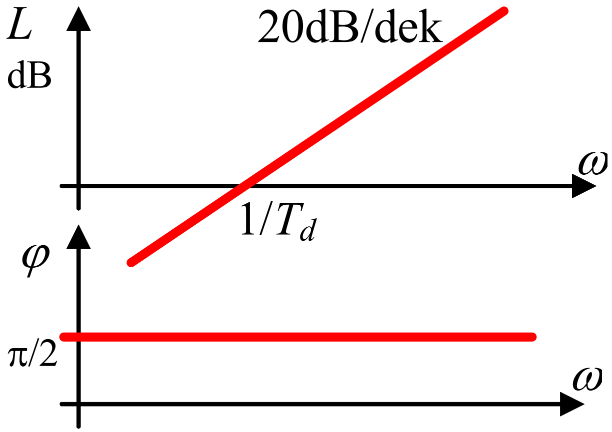


Logarytmiczne charakterystyki częstotliwościowe (podstawowe)

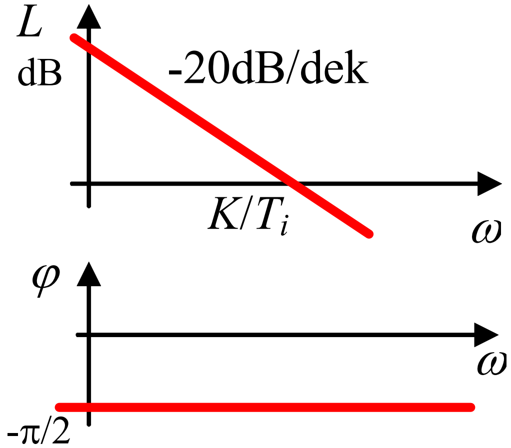
$$G(j\omega) = K$$



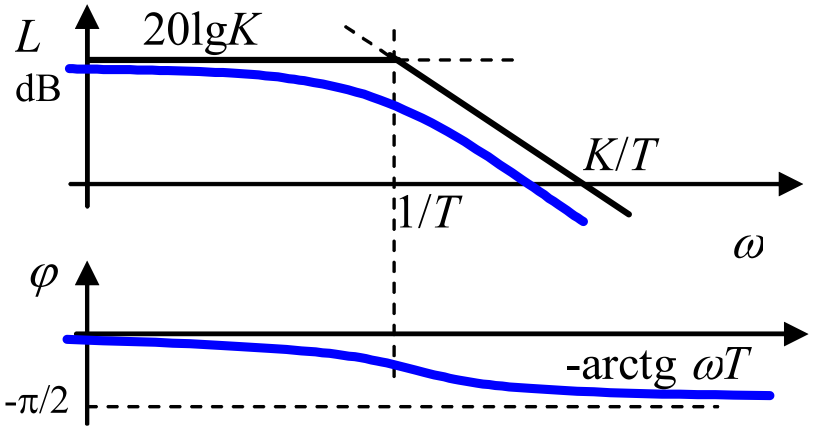
$$G(j\omega) = j\omega T_d$$



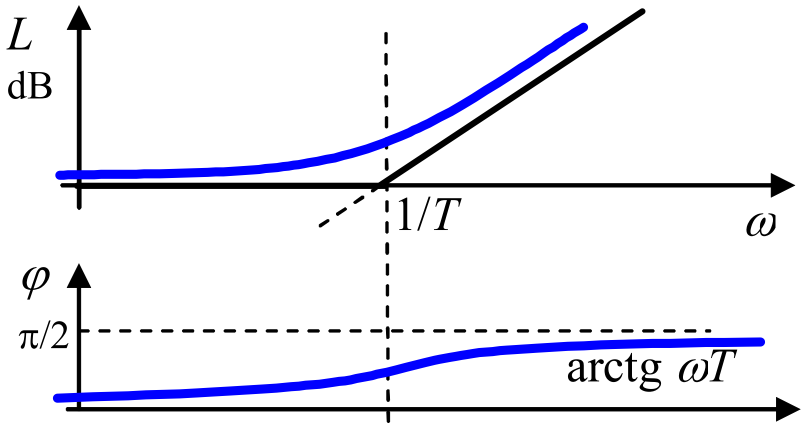
$$G(j\omega) = \frac{K}{j\omega T_i} = -j\frac{K}{\omega T_i}$$



$$G(j\omega) = \frac{K}{1 + j\omega T}$$

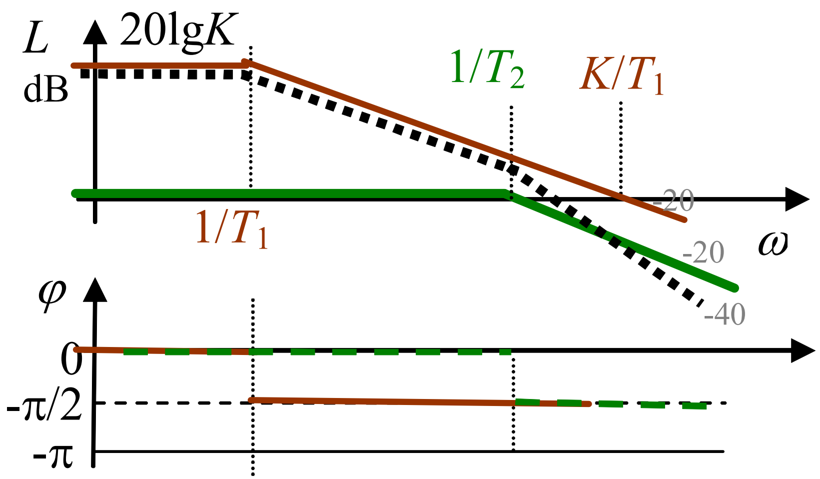


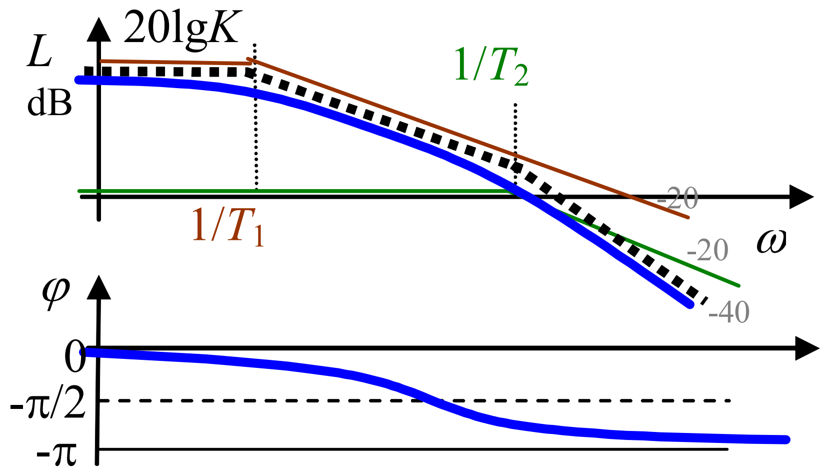
$$G(j\omega) = 1 + j\omega T$$



Logarytmiczne charakterystyki częstotliwościowe (złożone)

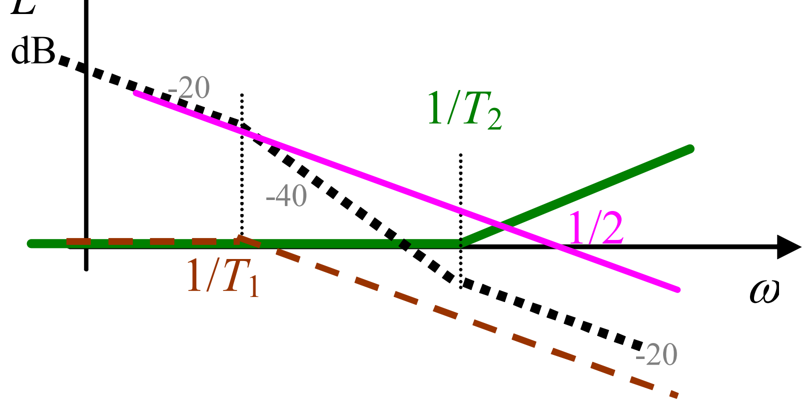
$$\frac{K}{(1+j\omega T_1)(1+j\omega T_2)},\, T_1 > T_2$$





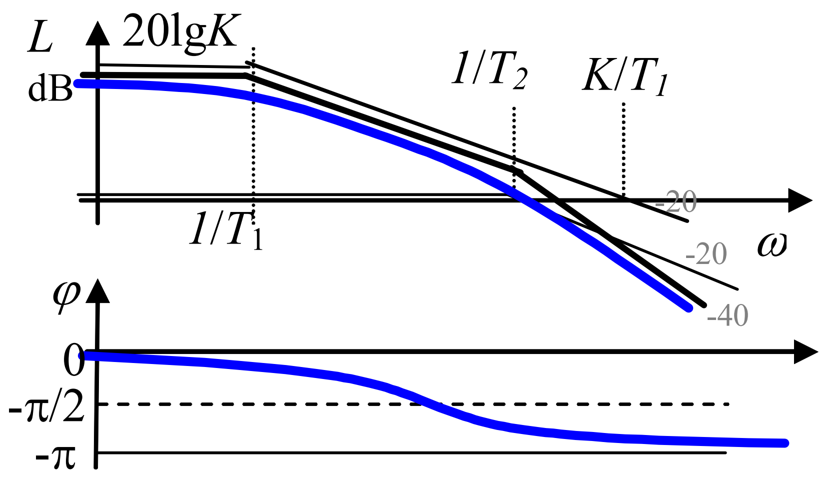
$$\frac{(1+j\omega T_2)}{2s(1+j\omega T_1)} = \frac{1}{2s}\frac{1}{(1+j\omega T_1)}(1+j\omega T_2)$$

Zał.: $T_1 > T_2 > (T_i{=}2)$

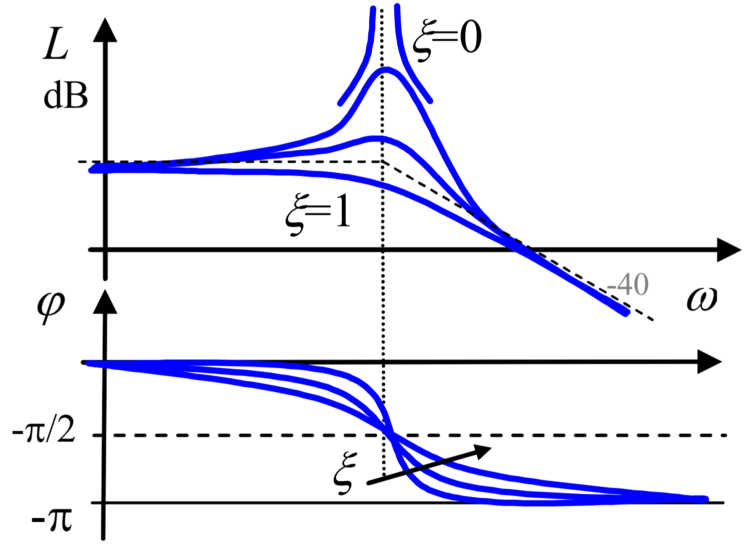


Logarytmiczne charakterystyki częstotliwościowe (szczególne)

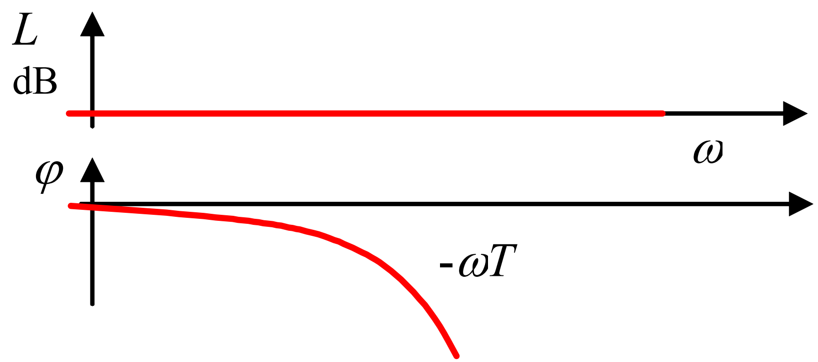
$$\frac{K}{1+j2\xi T\omega+(j\omega T)^2}=\frac{K}{(1+j\omega T_1)(1+j\omega T_2)},\xi>1$$



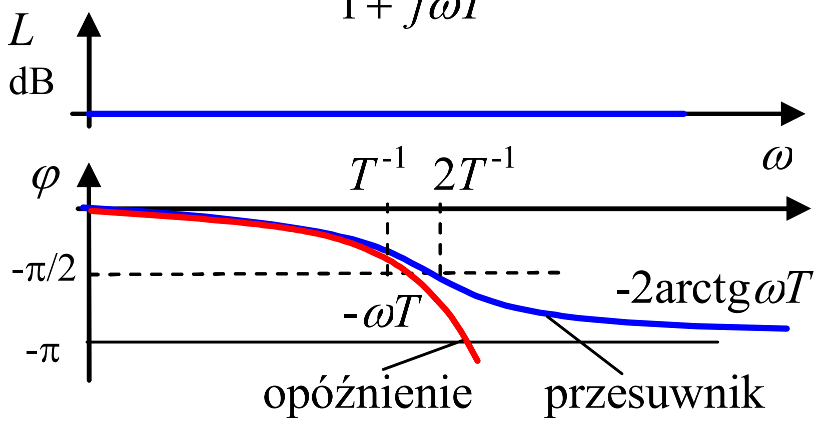
$$\frac{K}{1+j2\xi T\omega+(j\omega T)^2},\xi<1$$



$$G(j\omega)=e^{-j\omega T_o}$$

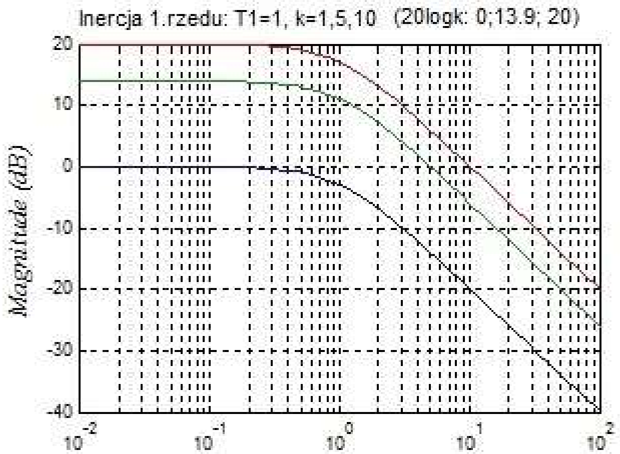


$$G(j\omega)=\frac{1-j\omega T}{1+j\omega T}$$

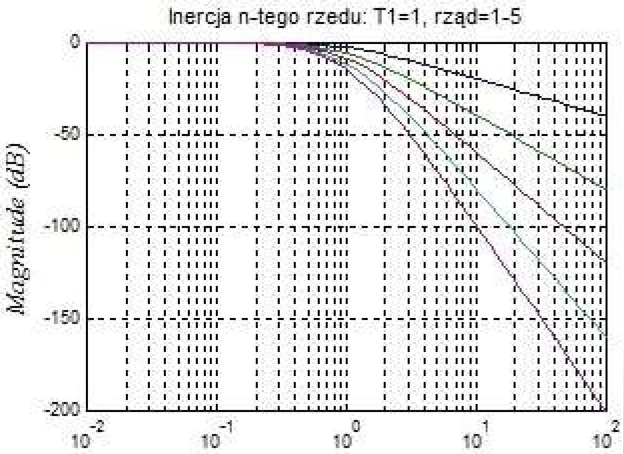


Charakterystyki częstotliwościowe – wpływ parametrów

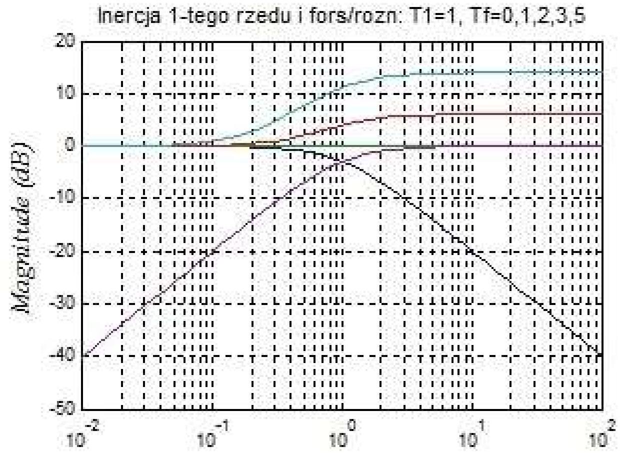
$$\frac{k}{(T_1 s+1)}$$

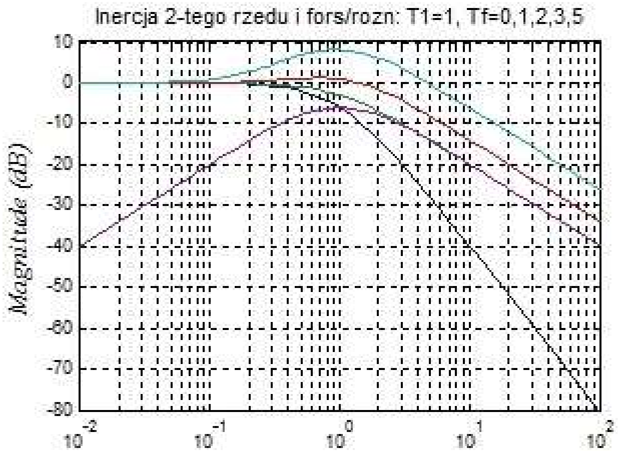


$$\frac{k}{(T_1 s+1)^n}$$

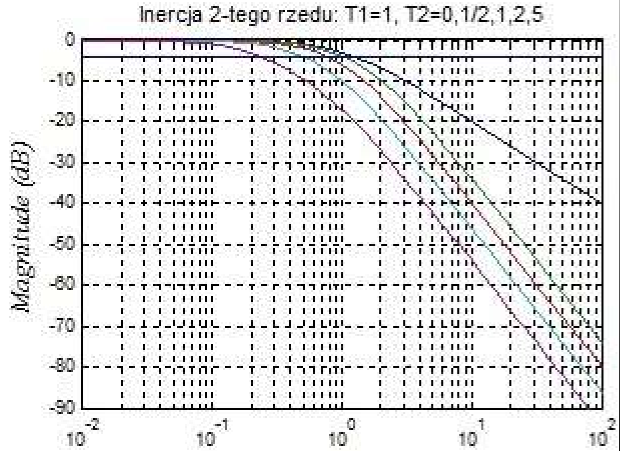


$$\frac{k(T_f s+1)}{(T_1 s+1)} \qquad \frac{ks}{(T_1 s+1)}$$





$$\frac{k(T_f s+1)}{(T_1 s+1)^2} \qquad \frac{ks}{(T_1 s+1)^2}$$



$$\frac{k}{(T_1 s+1)(T_2 s+1)}$$

$$\frac{k(T_f s+1)}{(T_1 s+1)(T_2 s+1)} \qquad \frac{ks}{(T_1 s+1)(T_2 s+1)}$$

Regulator PID – charakterystyki częstotliwościowe

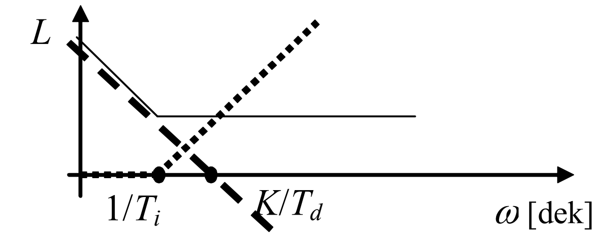
PI:

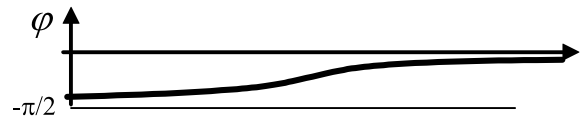
$$G(s) = K\left(1 + \frac{1}{sT_i}\right)$$

$$G(j\omega) = K - j\frac{K}{\omega T_i}$$

$$L(\omega) = 20\lg\left|K\frac{1 + j\omega T_i}{j\omega T}\right|$$

$$L(\omega) = 20\lg\left|\frac{K}{j\omega T}\right| + 20\lg\left|1 + j\omega T_i\right|$$





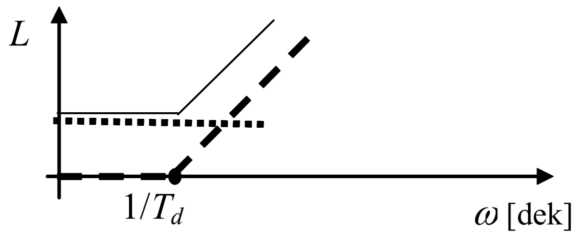
PD:

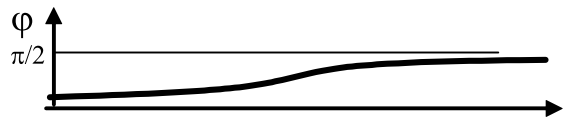
$$G(s) = K(1 + sT_d)$$

$$G(j\omega) = K + j\omega T_d$$

$$L(\omega) = 20\lg\left|K(1 + j\omega T_d)\right|$$

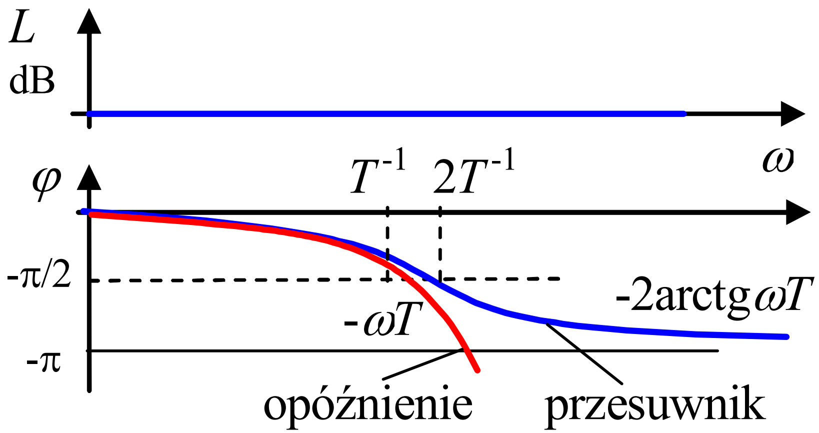
$$L(\omega) = 20\lg|K| + 20\lg|1 + j\omega T_d|$$





Logarytmiczne charakterystyki częstotliwościowe

układy minimalnofazowe

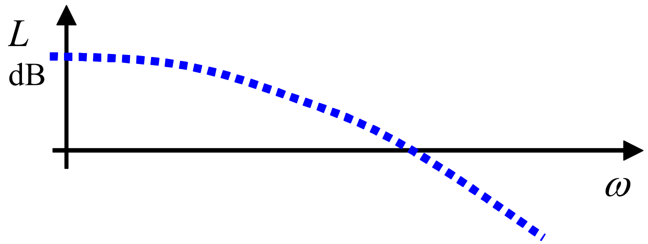


Własności logarytmicznych charakterystyk częstotliwościowych

- sumowanie charakterystyk dla członów połączonych szeregowo
- asymptoty charakterystyki amplitudowej – nachylenie +/- 20 dB/dek
- każdy biegun objawia się załamaniem asymptoty o -20 dB/dek
- każde zero objawia się załamaniem asymptoty o +20 dB/dek
- określony maksymalny błąd charakterystyk asymptotycznych członu inercyjnego i forsującego
    - dla częstości załamania popełnia się błąd 3dB
    - w odległości oktawy od częstości załamania – błąd 1 dB
- dla układów minimalnofazowych można otworzyć ch. fazową

Zdejmowanie charakterystyk częstotliwościowych

Opis eksperymentu



Wymuszenie: $u(t) = \sin \omega t$

Rozwiązanie: $x(t) = \dfrac{b\omega}{b^2\omega^2 + c^2} e^{-(c/b)t} + 1/\sqrt{b^2\omega^2 + c^2}\,\sin\left(\omega t - \mathrm{arctg}\dfrac{b\omega}{c}\right)$

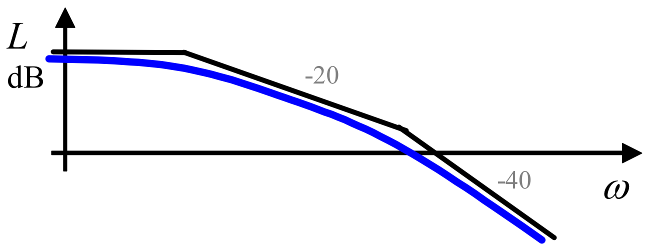
Stan ustalony: $x(t) = 1/\sqrt{b^2\omega^2 + c^2}\,\sin\left(\omega t - \mathrm{arctg}\dfrac{b\omega}{c}\right)$

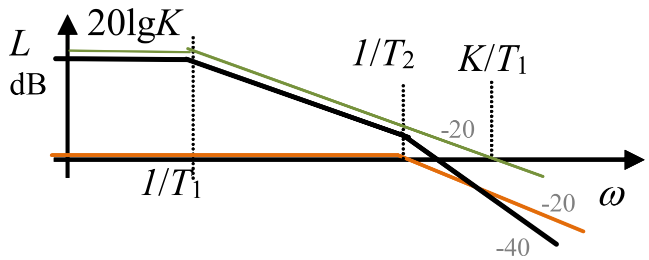
$$x(t) = A(\omega)\sin(\omega t - \varphi(\omega))$$

Wzmocnienie: $M(\omega) = \dfrac{A(\omega)}{1}$

$$\boxed{L(\omega) = 20\,\lg M(\omega)}$$

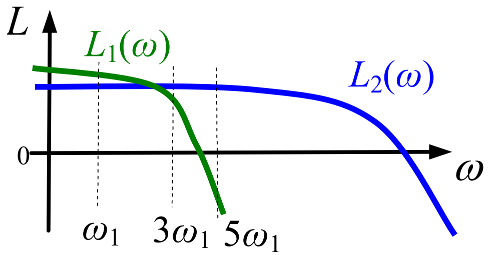
Identyfikacja modelu na podstawie charakterystyk częstotliwościowych

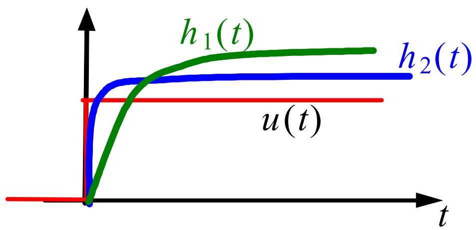


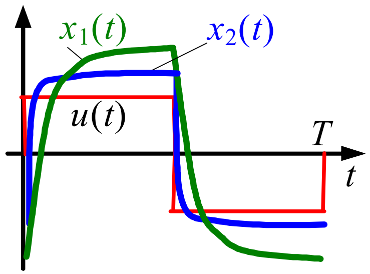


$$\frac{K}{(1 + j\omega T_1)} \cdot \frac{1}{(1 + j\omega T_2)}$$

Interpretacja

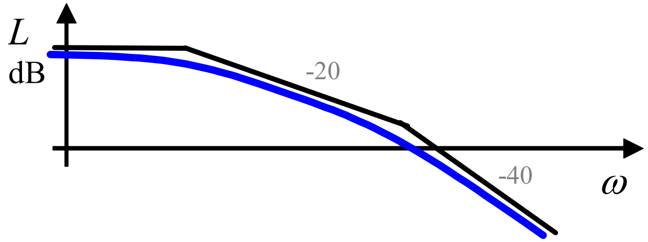


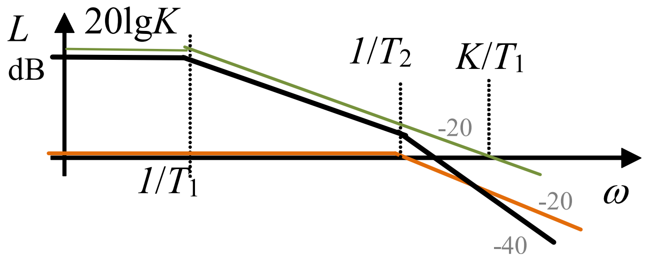




Zastosowanie charakterystyk częstotliwościowych

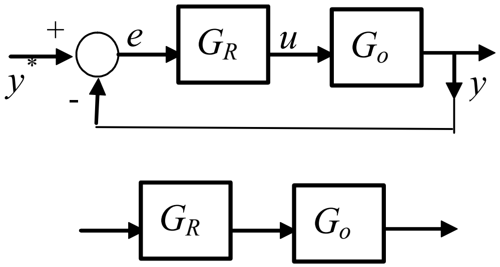
- Identyfikacja modelu na podstawie charakterystyk częstotliwościowych

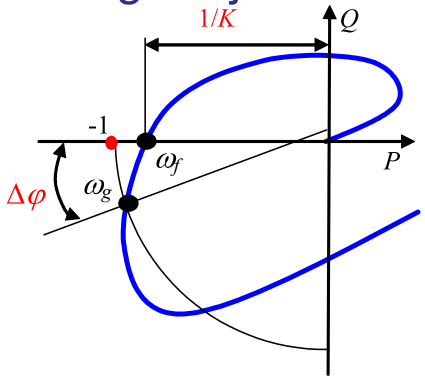


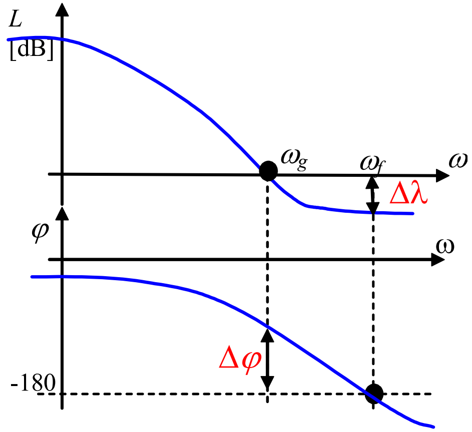


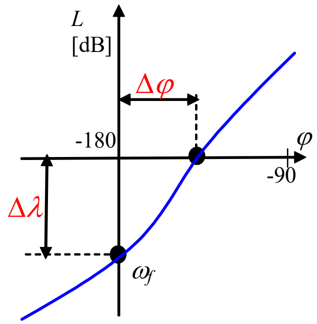
$$\frac{K}{(1+j\omega T_1)}\cdot\frac{1}{(1+j\omega T_2)}$$

- Kryterium stabilności układu regulacji









- Projektowanie filtrów
- Korekcja własności dynamicznych
  - pasmo przenoszenia
  - kompensacja biegunów