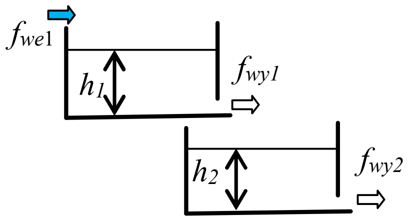
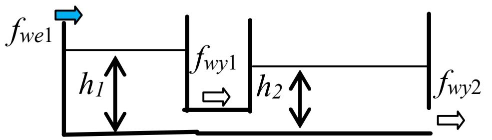
Układy równań różniczkowych

Kaskady

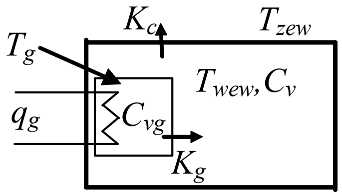


$$\begin{cases} A_1\dot{h}_1(t) = f_{we1}(t) - a_1 h_1(t) \\ A_2\dot{h}_2(t) = a_1 h_1(t) - a_2 h_2(t) \end{cases}$$

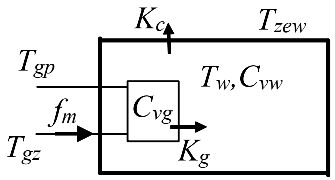


$$\begin{cases} A_1\dot{h}_1(t) = f_{we}(t) - a_1\big(h_1(t) - h_2(t)\big) \\ A_2\dot{h}_2(t) = a_1\big(h_1(t) - h_2(t)\big) - a_2 h_2(t) \end{cases}$$

Układy cieplne

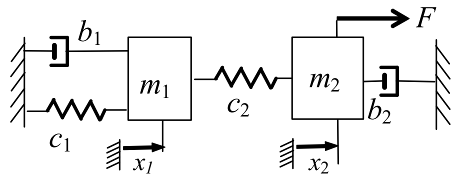


$$\begin{cases} C_{vg}\dot{T}_{gp}(t) = q_g(t) - K_g\big(T_g(t) - T_{wew}(t)\big) \\ C_{vw}\dot{T}_{wew}(t) = K_g\big(T_g(t) - T_{wew}(t)\big) - K_c\big(T_{wew}(t) - T_{zew}(t)\big) \end{cases}$$



$$\begin{cases} C_{vg}\dot{T}_{gp}(t) = c_{pw} f_m(t)\big(T_{gz}(t) - T_{gp}(t)\big) - K_g\big(T_{gp}(t) - T_{wew}(t)\big) \\ C_{vw}\dot{T}_{wew}(t) = K_g\big(T_{gp}(t) - T_{wew}(t)\big) - K_c\big(T_{wew}(t) - T_{zew}(t)\big) \end{cases}$$

Układy mechaniczne



$$\begin{cases} F(t) = m_2\ddot{x}_2(t) + b_2\dot{x}_2(t) + c_2\big(x_2(t) - x_1(t)\big) \\ 0 = m_1\ddot{x}_1(t) + b_1\dot{x}_1(t) + c_1 x_1(t) + c_2\big(x_1(t) - x_2(t)\big) \end{cases}$$

Równania stanu

$$\begin{cases} \dot{x}_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + b_{11}u_1 + b_{12}u_2 \\ \dot{x}_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + b_{21}u_1 + b_{22}u_2 \\ \dot{x}_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + b_{31}u_1 + b_{32}u_2 \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$ równania stanu

$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$ równania wyjściowe (równanie obserwacji)

u – wektor wejść
x – wektor stanu
y – wektor wyjść (wektor obserwacji), bo:

- nie wszystkie składowe **x** są przedmiotem zainteresowania
- bywa, że nie wszystkie składowe **x** są mierzalne (obserwowalne)

Najczęściej **D** = [0], bo nie ma bezpośredniego oddziaływania we na wy

Równania stanu - stabilność

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix} = 0$$

Równanie charakterystyczne

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

$$|\mathbf{A} - \lambda\mathbf{I}| = 0$$

Równanie n-tego rzędu → układ n równań 1-ego rzędu

$$a_3\dddot{x}(t)+a_2\ddot{x}(t)+a_1\dot{x}(t)+a_0x(t)=u(t)$$

$$a_3\lambda^3+a_2\lambda^2+a_1\lambda+a_0=0$$

rząd ?

$$x = x_1$$

$$\dot{x}_1 = x_2 \qquad = \dot{x}$$

$$\dot{x}_2 = x_3 \qquad = \ddot{x}$$

$$\dot{x}_3 \qquad = \dddot{x}$$

$$\dot{x}_k = x_{k+1}$$

(zmienne fazowe)

$$a_3\dot{x}_3(t)+a_2x_3(t)+a_1x_2(t)+a_0x_1(t)=u(t)$$

$$\begin{cases}\dot{x}_1(t)=x_2(t)\\ \dot{x}_2(t)=x_3(t)\\ \dot{x}_3(t)=\left[u(t)-a_0x_1(t)-a_1x_2(t)-a_2x_3(t)\right]/a_3\end{cases}$$

rząd ?

$$\begin{vmatrix}0-\lambda & 1 & 0\\ 0 & 0-\lambda & 1\\ \frac{-a_0}{a_3} & \frac{-a_1}{a_3} & \frac{-a_2}{a_3}-\lambda\end{vmatrix}=0$$

$$a_3\lambda^3+a_2\lambda^2+a_1\lambda+a_0=0$$

stopień ?

Badanie układów liniowych: 1) punkt równowagi

(składowa wymuszona przy stałym wymuszeniu)

$$a_n x^{(n)} + a_{n-1} x^{(n-1)} + \ldots + a_1 \dot{x} + a_0 x = b_m u^{(m)} + a_{m-1} u^{(m-1)} + \ldots + b_1 \dot{u} + b_0 u$$

$$a_0 x = b_0 u$$

$$\begin{cases} \dot{x}_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + b_{11}u_1 + b_{12}u_2 \\ \dot{x}_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + b_{21}u_1 + b_{22}u_2 \\ \dot{x}_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + b_{31}u_1 + b_{32}u_2 \end{cases}$$

$$\begin{cases} 0 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + b_{11}u_1 + b_{12}u_2 \\ 0 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + b_{21}u_1 + b_{22}u_2 \\ 0 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + b_{31}u_1 + b_{32}u_2 \end{cases}$$

$x_1 =$

$x_2 =$

$x_3 =$

Macierzowo:

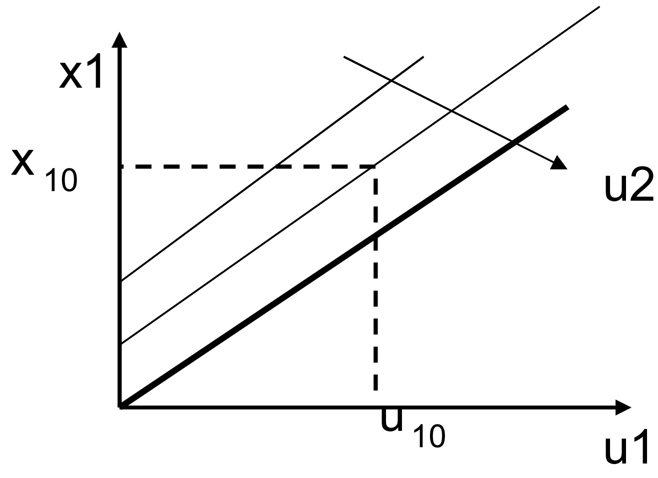
$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$

$$0 = \mathbf{Ax} + \mathbf{Bu}$$

$$\mathbf{Ax} = -\mathbf{Bu}$$

$$\mathbf{x} = -\mathbf{A}^{-1}\mathbf{Bu}$$

charakterystyka statyczna
(punkt równowagi)



Badanie układów liniowych: 2) stabilność

(składowa swobodna)

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix} = 0$$

Rozwiązanie swobodne:
- równanie charakterystyczne -> pierwiastki (bieguny układu)
- suma przebiegów wykładniczych
- przebiegi sin/cos z nałożonym przebiegiem wykładniczym
- układ jest stabilny jeśli $Re(\lambda_i) < 0$

Równanie charakterystyczne – własności wielomianu

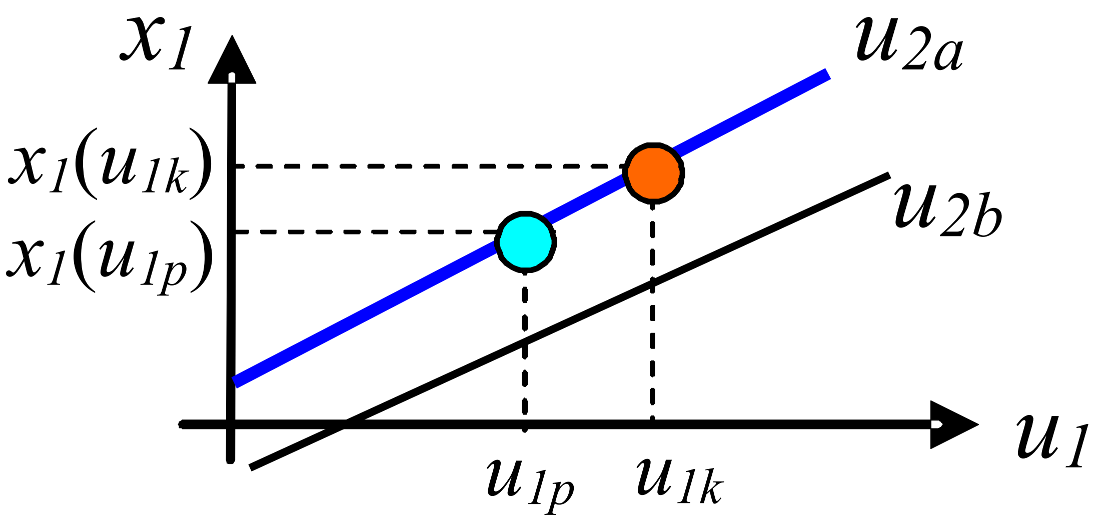
↓

kryteria położenia pierwiastków

↓

kryteria stabilności

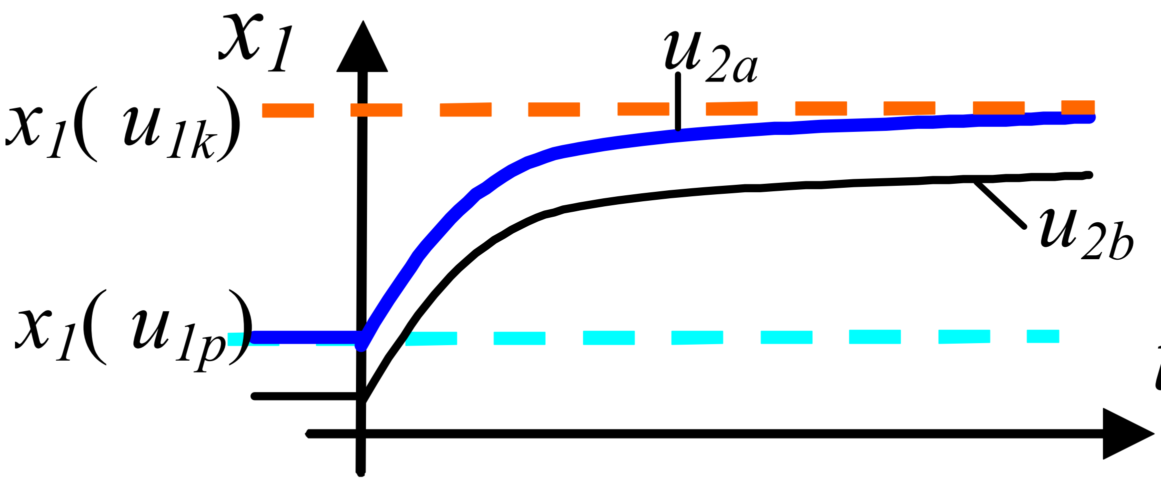
Charakterystyki statyczne i dynamiczne (czasowe)



$$x_1 = f\left(u_1, u_2\right)$$

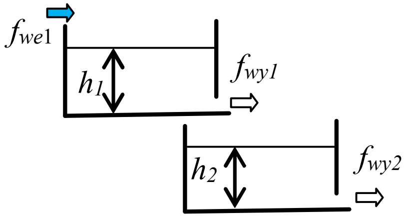
$$x_1 = f_1\left(u_1\right); \; u_2 = u_{2a}$$

$$x_1 = f_2\left(u_1\right); \; u_2 = u_{2b}$$



Odpowiedzi na skok/implus: a) stan ustalony, b) skok na 1 wejście

Kaskada niewspółdziałająca

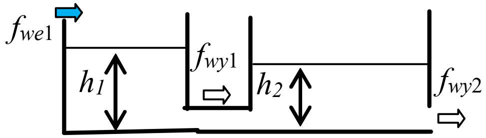


$$f_{wy1}(t) = a_1 h_1(t)$$
$$f_{wy2}(t) = a_2 h_2(t)$$

$$\begin{cases} A_1 \dot{h}_1(t) = f_{we1}(t) - a_1 h_1(t) \\ A_2 \dot{h}_2(t) = a_1 h_1(t) - a_2 h_2(t) \end{cases}$$

$$\begin{bmatrix} \dot{h}_1(t) \\ \dot{h}_2(t) \end{bmatrix} = \begin{bmatrix} \frac{-a_1}{A_1} & 0 \\ \frac{a_1}{A_2} & \frac{-a_2}{A_2} \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix} [f_{we1}(t)]$$

Kaskada współdziałająca



$$f_{wy1}(t) = a_1 (h_1(t) - h_2(t))$$
$$f_{wy2}(t) = a_2 h_2(t)$$

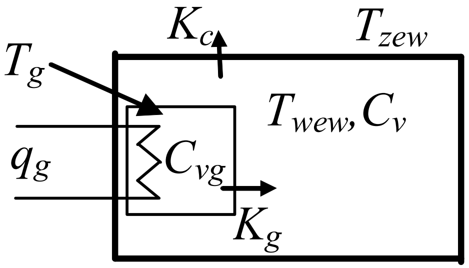
$$\begin{cases} A_1 \dot{h}_1(t) = f_{we}(t) - a_1 (h_1(t) - h_2(t)) \\ A_2 \dot{h}_2(t) = a_1 (h_1(t) - h_2(t)) - a_2 h_2(t) \end{cases}$$

$$\begin{bmatrix} \dot{h}_1(t) \\ \dot{h}_2(t) \end{bmatrix} = \begin{bmatrix} \frac{-a_1}{A_1} & \frac{a_1}{A_1} \\ \frac{a_1}{A_2} & \frac{-a_1 - a_2}{A_2} \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix} [f_{we1}(t)]$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

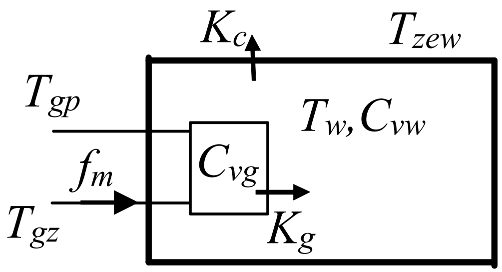
$$\begin{bmatrix} f_{wy1}(t) \\ f_{wy2}(t) \end{bmatrix} = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [f_{we1}(t)]$$

$$\begin{bmatrix} f_{wy1}(t) \\ f_{wy2}(t) \end{bmatrix} = \begin{bmatrix} a_1 & -a_1 \\ 0 & a_2 \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [f_{we1}(t)]$$



$$\begin{cases} C_{vg}\dot{T}_{gp}(t) = q_g(t) - K_g\left(T_g(t) - T_{wew}(t)\right) \\ C_{vw}\dot{T}_{wew}(t) = K_g\left(T_g(t) - T_{wew}(t)\right) - K_c\left(T_{wew}(t) - T_{zew}(t)\right) \end{cases}$$

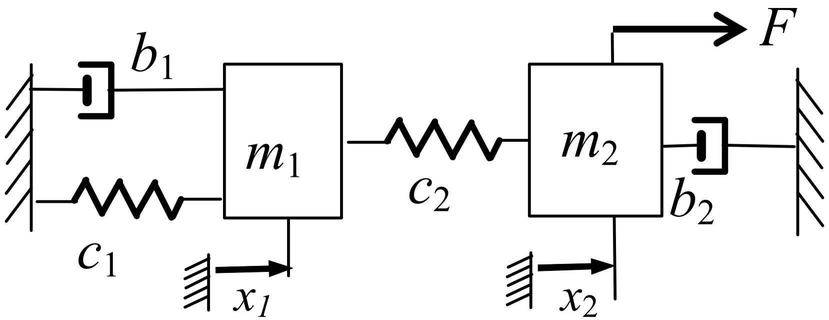
$$\begin{bmatrix} \dot{T}_g(t) \\ \dot{T}_{wew}(t) \end{bmatrix} = \begin{bmatrix} \frac{-K_g}{C_{vg}} & \frac{K_g}{C_{vg}} \\ \frac{K_g}{C_{vw}} & \frac{-K_g - K_c}{C_{vw}} \end{bmatrix} \begin{bmatrix} T_g(t) \\ T_{wew}(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{C_{vg}} & 0 \\ 0 & \frac{K_c}{C_{vw}} \end{bmatrix} \begin{bmatrix} q_g(t) \\ T_{zew}(t) \end{bmatrix}$$



$$\begin{cases} C_{vg}\dot{T}_{gp}(t) = c_{pw} f_m(t)\left(T_{gz}(t) - T_{gp}(t)\right) - K_g\left(T_{gp}(t) - T_{wew}(t)\right) \\ C_{vw}\dot{T}_{wew}(t) = K_g\left(T_{gp}(t) - T_{wew}(t)\right) - K_c\left(T_{wew}(t) - T_{zew}(t)\right) \end{cases}$$

$$f_m(t) = f_{m0} = const$$

$$\begin{bmatrix} \dot{T}_{gp}(t) \\ \dot{T}_{wew}(t) \end{bmatrix} = \begin{bmatrix} \frac{-c_{pw} f_{m0} - K_g}{C_{vg}} & \frac{K_g}{C_{vg}} \\ \frac{K_g}{C_{vw}} & \frac{-K_g - K_c}{C_{vw}} \end{bmatrix} \begin{bmatrix} T_{gp}(t) \\ T_{wew}(t) \end{bmatrix} + \begin{bmatrix} \frac{c_{pw} f_{m0}}{C_{vg}} & 0 \\ 0 & \frac{K_c}{C_{vw}} \end{bmatrix} \begin{bmatrix} T_{gz}(t) \\ T_{zew}(t) \end{bmatrix}$$



$$\begin{cases} F(t) = m_2\ddot{x}_2(t) + b_2\dot{x}_2(t) + c_2\left(x_2(t) - x_1(t)\right) \\ 0 = m_1\ddot{x}_1(t) + b_1\dot{x}_1(t) + c_1x_1(t) + c_2\left(x_1(t) - x_2(t)\right) \end{cases}$$

Konwersja:

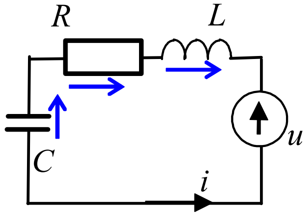
$\dot{x}_1 = x_{1a} \qquad x_1 = x_{1b}\ (\dot{x}_{1b} = x_{1a})$

$\dot{x}_2 = x_{2a} \qquad x_2 = x_{2b}\ (\dot{x}_{2b} = x_{2a})$

$$\begin{bmatrix} \dot{x}_{1a}(t) \\ \dot{x}_{2a}(t) \\ \dot{x}_{1b}(t) \\ \dot{x}_{2b}(t) \end{bmatrix} = \begin{bmatrix} \frac{-b_1}{m_1} & 0 & \frac{-c_1-c_2}{m_1} & \frac{c_2}{m_1} \\ 0 & \frac{-b_2}{m_2} & \frac{c_2}{m_2} & \frac{-c_2}{m_2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1a}(t) \\ x_{2a}(t) \\ x_{1b}(t) \\ x_{2b}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m_2 \\ 0 \\ 0 \end{bmatrix} F(t)$$

$$m_1m_2\lambda^4 + (m_1b_2 + m_2b_1)\lambda^3 + \left(m_1c_2 + m_2(c_1+c_2) + b_1b_2\right)\lambda^2 + \left(b_1c_2 + b_2(c_1+c_2)\right)\lambda + c_1c_2 = 0$$

Proste obwody elektryczne (1)



$$L\frac{di(t)}{dt} + Ri(t) + \int \frac{i(t)}{C}dt = u(t) \qquad\qquad L\frac{di(t)}{dt} + R\dot{q}(t) + \frac{q(t)}{C} = u(t)$$

1) Wprowadzenie definicji natężenia prądu:

$$\dot{q}(t) = i(t) \quad \rightarrow \quad L\ddot{q}(t) + R\dot{q}(t) + \frac{q(t)}{C} = u(t)$$

$$\begin{array}{ll} q(t) & \rightarrow q(t) \\ \dot{q}(t) = i(t) & \rightarrow \dot{q}(t) \\ \dot{i}(t) & \rightarrow \ddot{q}(t) \end{array}$$

$$\begin{array}{llll} x = & x_1 & & \\ & \dot{x}_1 = & x_2 & = \dot{x} \\ & & \dot{x}_2 = \ x_3 & = \ddot{x} \\ \text{(slajd 3)} & & \dot{x}_3 & = \dddot{x} \end{array}$$

$$\begin{cases} L\dfrac{di(t)}{dt} + Ri(t) + \dfrac{q(t)}{C} = u(t) \\ \dfrac{dq(t)}{dt} = i(t) \end{cases} \qquad \begin{bmatrix} \dot{i}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} -R/L & 1/(LC) \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} u(t)$$

$u_C(t) = q(t)/C$
(r.wyjściowe)

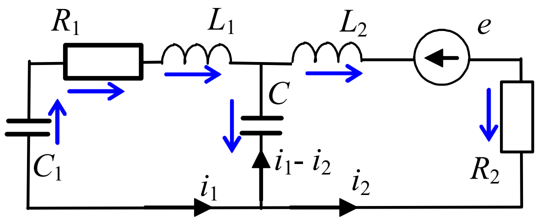
2) Zmienne stanu = niezależne napięcia na kondensatorach i natężenia prądów w cewkach:

$$u_C(t) = \int \frac{i(t)}{C}dt \quad \rightarrow C\frac{u_C(t)}{dt} = i(t)$$

$$\begin{cases} L\dfrac{di(t)}{dt} + Ri(t) + u_C(t) = u(t) \\ C\dfrac{du_C(t)}{dt} = i(t) \end{cases} \qquad \begin{bmatrix} \dot{i}(t) \\ \dot{u}_C(t) \end{bmatrix} = \begin{bmatrix} -R/L & -1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} i(t) \\ u_C(t) \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} u(t)$$

$q(t) = Cu_C(t)$
(r.wyjściowe)

Proste obwody elektryczne (2)



$$\begin{cases} 0 = L_1 \dfrac{di_1(t)}{dt} + R_1 i_1(t) + \displaystyle\int \dfrac{i_1(t) - i_2(t)}{C} dt + \int \dfrac{i_1(t)}{C_1} dt \\ e(t) = L_2 \dfrac{di_2(t)}{dt} + R_2 i_2(t) + \displaystyle\int \dfrac{i_2(t) - i_1(t)}{C} dt \end{cases}$$

1) Wprowadzenie definicji natężenia prądu:

$$\dot{q}_1 = i_1 \qquad q_1 = q_{1a} \quad (\dot{q}_{1a} = i_1)$$

$$\dot{q}_2 = i_2 \qquad q_2 = q_{2a} \quad (\dot{q}_{2a} = i_2)$$

$$\begin{cases} 0 = L_1 \ddot{q}_1(t) + R_1 \dot{q}_1(t) + \dfrac{q_1(t) - q_2(t)}{C} + \dfrac{q_1(t)}{C_1} \\ e = L_2 \ddot{q}_2(t) + R_2 \dot{q}_2(t) + \dfrac{q_2(t) - q_1(t)}{C} \end{cases}$$

Dla $\mathbf{x}(t) = [i_1(t), i_2(t), q_1(t), q_2(t)]^T$, $\mathbf{u}(t) = e(t)$

$$\mathbf{A} = \begin{bmatrix} \dfrac{-R_1}{L_1} & 0 & \dfrac{-(C_1 + C)}{L_1 C_1 C} & \dfrac{1}{L_1 C} \\ 0 & \dfrac{-R_2}{L_2} & \dfrac{1}{L_2 C} & \dfrac{-1}{L_2 C} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ \dfrac{1}{L_2} \\ 0 \\ 0 \end{bmatrix}$$

$$L_1 L_2 C \lambda^4 + (L_1 R_2 + L_2 R_1) C \lambda^3 + (L_1 + L_2 + R_1 R_2 C) \lambda^2 + (R_1 + R_2) \lambda = 0$$

2) Zmienne stanu = niezależne napięcia na kondensatorach i natężenia prądów w cewkach:

$$u_C(t) = \int \frac{i_1(t) - i_2(t)}{C} dt \qquad u_{C1}(t) = \int \frac{i_1(t)}{C_1} dt,$$

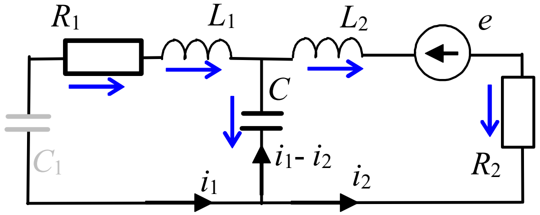
$$\left(C \dot{u}_C(t) = i_1(t) - i_2(t) \qquad C_1 \dot{u}_{C1}(t) = i_1(t) \right)$$

$$\begin{cases} 0 = L_1 \dfrac{di_1(t)}{dt} + R_1 i_1(t) + u_C(t) + u_{C1}(t) \\ e(t) = L_2 \dfrac{di_2(t)}{dt} + R_2 i_2(t) - u_C(t) \end{cases}$$

Dla $\mathbf{x}(t) = [i_1(t), i_2(t), u_C(t), u_{C1}(t)]^T$, $\mathbf{u}(t) = e(t)$

$$\mathbf{A} = \begin{bmatrix} \dfrac{-R_1}{L_1} & 0 & \dfrac{-1}{L_1} & \dfrac{-1}{L_1} \\ 0 & \dfrac{-R_2}{L_2} & \dfrac{1}{L_2} & 0 \\ \dfrac{1}{C} & \dfrac{-1}{C} & 0 & 0 \\ \dfrac{1}{C_1} & 1 & 0 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ \dfrac{1}{L_2} \\ 0 \\ 0 \end{bmatrix}$$

$$L_1 L_1 C \lambda 3^2 + (L_1 R_2 + L_2 R_1) C \lambda^3 + (L_1 + L_2 + R_1 R_2 C) \lambda + R_1 + R_2 = 0$$



$$\begin{cases} 0 = L_1 \dfrac{di_1(t)}{dt} + R_1 i_1(t) + \displaystyle\int \dfrac{i_1(t) - i_2(t)}{C} dt \\ e(t) = L_2 \dfrac{di_2(t)}{dt} + R_2 i_2(t) + \displaystyle\int \dfrac{i_2(t) - i_1(t)}{C} dt \end{cases}$$

1) Wprowadzenie definicji natężenia prądu:

$$\dot{q}_1 = i_1 \qquad q_1 = q_{1a} \quad (\dot{q}_{1a} = i_1)$$

$$\dot{q}_2 = i_2 \qquad q_2 = q_{2a} \quad (\dot{q}_{2a} = i_2)$$

$$\begin{cases} 0 = L_1 \ddot{q}_1(t) + R_1 \dot{q}_1(t) + \dfrac{q_1(t) - q_2(t)}{C} \\ e = L_2 \ddot{q}_2(t) + R_2 \dot{q}_2(t) + \dfrac{q_2(t) - q_1(t)}{C} \end{cases}$$

Dla $\mathbf{x}(t) = [i_1(t), i_2(t), q_1(t), q_2(t)]^T$, $\mathbf{u}(t) = e(t)$

$$\mathbf{A} = \begin{bmatrix} \dfrac{-R_1}{L_1} & 0 & \dfrac{-1}{L_1 C} & \dfrac{1}{L_1 C} \\ 0 & \dfrac{-R_2}{L_2} & \dfrac{1}{L_2 C} & \dfrac{-1}{L_2 C} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 \\ \dfrac{1}{L_2} \\ 0 \\ 0 \end{bmatrix}$$

2) Zmienne stanu = niezależne napięcia na kondensatorach i natężenia prądów w cewkach:

$$u_C(t) = \int \frac{i_1(t) - i_2(t)}{C} dt \qquad u_{C1}(t) = \int \frac{i_1(t)}{C_1} dt,$$

$$\left(C\dot{u}_C(t) = i_1(t) - i_2(t) \qquad C_1 \dot{u}_{C1}(t) = i_1(t) \right)$$

Dla $\mathbf{x}(t) = [i_1(t), i_2(t), u_C(t)]^T$, $\mathbf{u}(t) = e(t)$

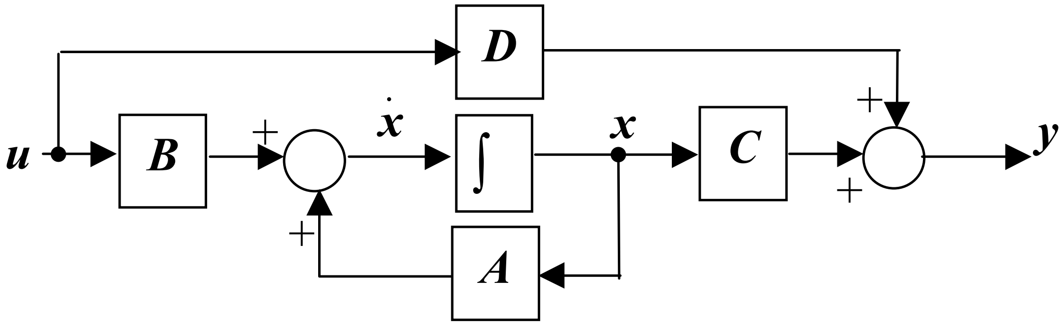
$$\begin{cases} 0 = L_1 \dfrac{di_1(t)}{dt} + R_1 i_1(t) + u_C(t) \\ e(t) = L_2 \dfrac{di_2(t)}{dt} + R_2 i_2(t) - u_C(t) \end{cases}$$

$$\mathbf{A} = \begin{bmatrix} \dfrac{-R_1}{L_1} & 0 & \dfrac{-1}{L_1} \\ 0 & \dfrac{-R_2}{L_2} & \dfrac{1}{L_2} \\ \dfrac{1}{C} & \dfrac{-1}{C} & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 \\ \dfrac{1}{L_2} \\ 0 \\ 0 \end{bmatrix}$$

Równania stanu

$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ Równania stanu

$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$ Równania wyjściowe



$0 = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ Równania statyczne

$\mathbf{x} = -\mathbf{A}^{-1}\mathbf{B}\mathbf{u}$ Punkt równowagi

$|\mathbf{A} - \lambda\mathbf{I}| = 0$ Równanie charakterystyczne

$|s\mathbf{I} - \mathbf{A}| = 0$

λ – wartości własne macierzy $\mathbf{A}$

Stabilność