Modele złożone (MIMO, opóźnienia)

Modele:
- Pomieszczenie z grzejnikiem c.o
- Pomieszczenie z grzejnikiem + kotłownia
- 3*pomieszczenie z grzejnikiem + kotłownia

+ *regulacja* T_{wew}

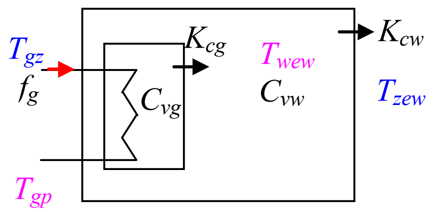
Założenie: $f_g = const$

Zadania:
- Opis obiektu
- Identyfikacja parametrów (obliczenie współczynników)
- Obliczenie stanu równowagi obiektu – charakterystyka statyczna obiektu
- Badanie stabilności obiektu
- Obliczenie stanu równowagi układu regulacji PI
- Badanie stabilności układu – dobór nastaw

Funkcje:

ss, tf, zkp, series (*), parallel (+), feedback
pole, zero, pzmap
step, impulse, nyquist, bode
....

1a) Pomieszczenie z grzejnikiem c.o - równania różniczkowe obiektu



Z1: f_g=const

we: T_{gz}, T_{zew}; *wy:* T_{wew}, T_{gp}

$$\begin{cases} C_{vw}\dot{T}_{wew} = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ C_{vg}\dot{T}_{gp} = c_{pw}\rho_w f_g\left(T_{gz} - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \end{cases}$$

$$\begin{cases} 0 = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ 0 = c_{pw}\rho_w f_g\left(T_{gz} - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \end{cases}$$

$$c_{pw}\rho_w f_g\left(T_{gz} - T_{gp}\right) = K_{cg}\left(T_{gp} - T_{wew}\right) = K_{cw}\left(T_{wew} - T_{zew}\right)$$

Równania stanu: $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$

Funkcje: obiektS = ss(A, B, C, D)

$$\begin{bmatrix} \dot{T}_{wew} \\ \dot{T}_{gp} \end{bmatrix} = \begin{bmatrix} \dfrac{-K_{cg} - K_{cw}}{C_{vw}} & \dfrac{K_{cg}}{C_{vw}} \\ \dfrac{K_{cg}}{C_{vg}} & \dfrac{-c_{pw}\rho_w f_g - K_{cg}}{C_{vg}} \end{bmatrix} \begin{bmatrix} T_{wew} \\ T_{gp} \end{bmatrix} + \begin{bmatrix} 0 & \dfrac{K_{cw}}{C_{vw}} \\ \dfrac{c_{pw}\rho_w f_g}{C_{vg}} & 0 \end{bmatrix} \begin{bmatrix} T_{gz} \\ T_{zew} \end{bmatrix}$$

Bieguny (stabilność): wartości własne macierzy A

pole(obiektS)

Równania statyczne: $0 = \mathbf{Ax} + \mathbf{Bu}$

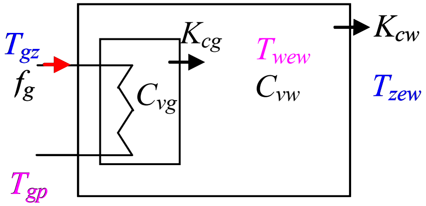
lub:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -K_{cg} - K_{cw} & K_{cg} \\ K_{cg} & -c_{pw}\rho_w f_g - K_{cg} \end{bmatrix} \begin{bmatrix} T_{wew} \\ T_{gp} \end{bmatrix} + \begin{bmatrix} 0 & K_{cw} \\ c_{pw}\rho_w f_g & 0 \end{bmatrix} \begin{bmatrix} T_{gz} \\ T_{zew} \end{bmatrix}$$

Punkt równowagi: $0 = \mathbf{Ax} + \mathbf{Bu} \Rightarrow \mathbf{Ax} = -\mathbf{Bu} \Rightarrow \mathbf{x} = -\mathbf{A}^{-1}\mathbf{Bu}$

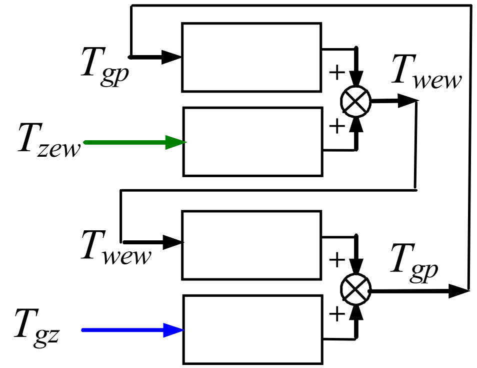
Badania podst.:
step(obiektG)
nyquist(obiektG)
bode(obiektG)
pzmap(obiektG)

1a) Pomieszczenie z grzejnikiem c.o – transmitancje obiektu



$$\begin{cases} C_{vw}\dot{T}_{wew} = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ C_{vg}\dot{T}_{gp} = c_{pw}\rho_w f_g\left(T_{gz} - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \end{cases}$$

Funkcje: obiektS= ss(A, B, C, D)
obiektG4 = tf(obiektS) ;obiektG4(wyj, wej)



Z1: f_g=const

we: T_{gz}, T_{zew}; wy: T_{wew}, T_{gp}

Transmitancje obiektu:

$$T_{wew} = \frac{c_{pw}\rho_w f_g K_{cg}}{M_1 M_2 - K_{cg}^2} T_{gz} + \frac{K_{cw} M_2}{M_1 M_2 - K_{cg}^2} T_{zew}$$

$$T_{gp} = \frac{c_{pw}\rho_w f_g M_1}{M_1 M_2 - K_{cg}^2} T_{gz} + \frac{K_{cg} K_{cw}}{M_1 M_2 - K_{cg}^2} T_{zew}$$

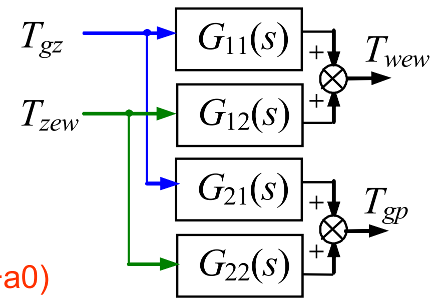
$$M_1 = C_{vw}s + K_{cg} + K_{cw}$$

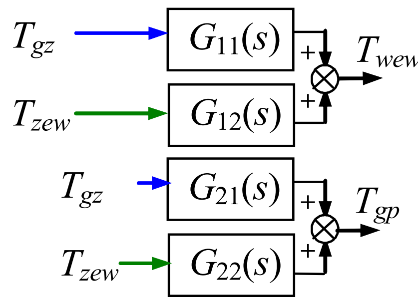
$$M_2 = C_{vg}s + K_{cg} + c_{pw}\rho_w f_g$$

$$T_{wew} = G_{11}T_{gz} + G_{12}T_{zew}$$

$$T_{gp} = G_{21}T_{gz} + G_{22}T_{zew}$$

Funkcje:
a) obiektG = tf([b1 b2], [a2, a1, a0])
b) s=tf('s'); obiektG = (b1*s+b0) / (a2^2*s+a1*s+a0)





Czy T_{wew} i T_{gp}, są niezależne?

Bieguny (stabilność): M(s)=0

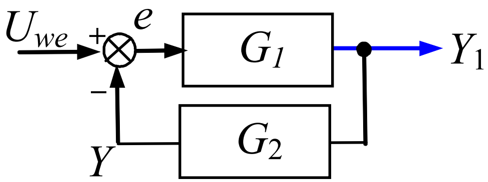
pole(obiektG)

Punkt równowagi:

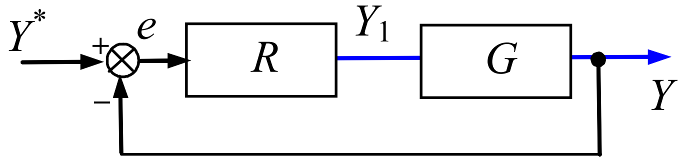
$$\lim_{t\to\infty} T_{wew}(t) = \lim_{s\to 0} sT_{wew}(s) = \lim_{s\to 0} s\left(G_{11}\frac{T_{gz0}}{s} + G_{12}\frac{T_{zew0}}{s}\right)$$

Badania podst.:
step(obiektG)
nyquist(obiektG)
bode(obiektG)
pzmap(obiektG)

Matlab: Transmitancje obiektu i układu regulacji



wyY1 = feedback(G1, G2)
wyY   = wyY1* G2



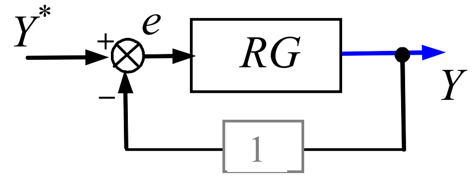
1) Gz = R*G / (1+R*G)

2) wyY1 = feedback(R, G)
    Gz = wyY1 * G

3) **Go = R *G**
    Gz = feedback(Go,1)

4) Gz = LR*LG / (1+ LR*LG)

$$G_z = \frac{Y}{Y^*} = \frac{RG}{1+RG}$$



$$R = \frac{L_R}{M_R}, \quad G = \frac{L_G}{M_G}$$

$$G_z = \frac{L_R L_G}{M_R M_G + L_R L_G}$$

Matlab: Transmitancje obiektu i układu regulacji - przykład

$$G = \frac{k}{Ts+1}, \qquad R = K_p\left(1+\frac{1}{T_i s}\right) = K_p\frac{T_i s+1}{T_i s}$$

```
s=tf('s');
k=2;    T1=2; To=1; Kp=1; Ti=1;
G= k/(T1*s+1);
R=Kp*(Ti*s + 1)/(Ti*s);
```

```
LG= k;
MG = T1*s+1;
LR=Kp*(Ti*s + 1);
MR = Ti*s;
```

	1) Gz = R*G / (1+R*G)	2) Gz=feedback(R, G) * G	3) **Gz = feedback(Rg*G,1)**	4) **Gz = LR*LG / (MR*MG+ LR*LG)**
Gz	(4 s^3 + 6 s^2 + 2 s) / (4 s^4 + 8 s^3 + 7 s^2 + 2 s)	(4 s^2 + 6 s + 2) / (4 s^3 + 8 s^2 + 7 s + 2)	(2 s + 2) / (2 s^2 + 3 s + 2)	(2 s + 2) / (2 s^2 + 3 s + 2)
zero(Gz)	0; **-1;** -0.5	**-1;** -0.5	**-1**	**-1**
pole(Gz)	0; **-0.75±0.6614i;** -0.5	**-0.75±0.6614i;** -0.5	**-0.75±0.6614i**	**-0.75±0.6614i**

$$G_z = \frac{RG}{1+RG} = \frac{\frac{L_R L_G}{M_R M_G}}{1+\frac{L_R L_G}{M_R M_G}} = \frac{\frac{L_R L_G}{M_R M_G}}{\frac{M_R M_G + L_R L_G}{M_R M_G}} = \frac{L_R L_G \cdot M_R M_G}{(M_R M_G + L_R L_G)\cdot M_R M_G} = \frac{L_R L_G}{M_R M_G + L_R L_G}$$

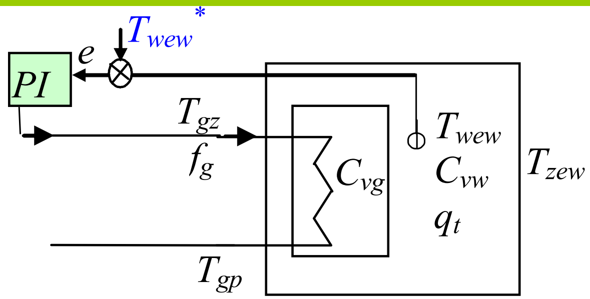
$$\frac{\frac{s+1}{s}\cdot\frac{2}{2s+1}}{1+\frac{s+1}{s}\cdot\frac{2}{2s+1}} \qquad \frac{2(s+1)\cdot s(2s+1)}{\left(s(2s+1)+2(s+1)\right)\cdot s(2s+1)}$$

$$\left(\frac{\frac{s+1}{s}}{1+\frac{s+1}{s}\cdot\frac{2}{2s+1}}\right)\cdot\frac{2}{2s+1} \qquad \left(\frac{2(s+1)\cdot \cancel{s}}{\left(s(2s+1)+2(s+1)\right)\cdot \cancel{s}}\right)\cdot\frac{2}{2s+1}$$

$$\left(\frac{\frac{2(s+1)}{s(2s+1)}}{1+\frac{2(s+1)}{s(2s+1)}}\right) \qquad \frac{2(s+1)}{s(2s+1)+2(s+1)}$$

$$\frac{(s+1)\cdot 2}{s(2s+1)+2(s+1)}$$

1a) Pomieszczenie z grzejnikiem c.o – transmitancje układu regulacji PI



Z1: f_g=const

we: T^*_{wew}, T_{zew}; wy: T_{wew}, T_{gp}, T_{gz}

Transmitancje obiektu

$$T_{wew} = G_{11}T_{gz} + G_{12}T_{zew}$$
$$T_{gp} = G_{21}T_{gz} + G_{22}T_{zew}$$

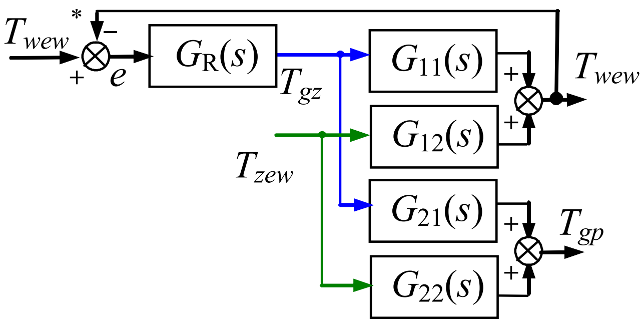
obiektG11 = ...
obiektG12 = ...
...

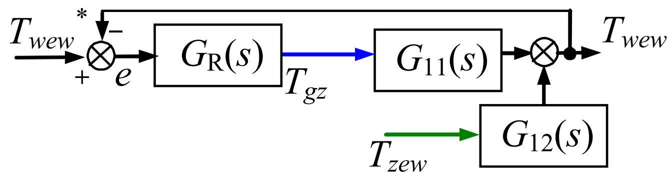
Transmitancja regulatora PI IND

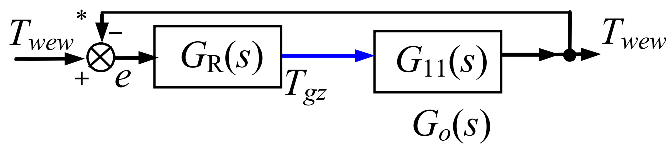
s=tf('s'); reg = Kp + KTi/s
;też funkcja parallel

$$T_{gz} = G_R e = \left(K_p + \frac{K_i}{s}\right)\left(T^*_{wew} - T_{wew}\right)$$

Transmitancje układu:







$$G_e = \frac{e}{T^*_{wew}} = \frac{1}{1 + G_R G_o}$$

$$G_z = \frac{T_{wew}}{T^*_{wew}} = \frac{G_R G_o}{1 + G_R G_o}$$

ukladGotw = reg*obiektG11
ukladGz = feedback(reg*obiektG11, 1)

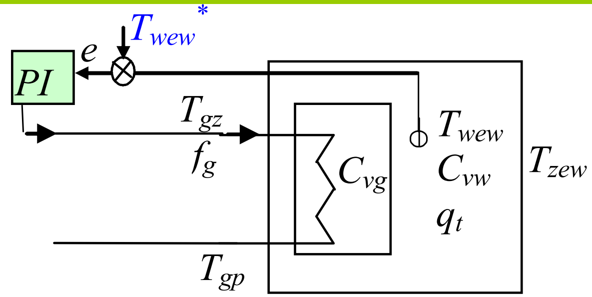
Bieguny (stabilność): $M_{Gz}(s)=0$

pole(ukladGz)

Punkt równowagi: $T_{wew} = T^*_{wew}$

Badania podst.:
step(ukladGz)
nyquist(ukladGotw)
bode(ukladGotw)
bode(ukladGz)
pzmap(ukladGz)

1b) Pomieszczenie z grzejnikiem c.o – r.różniczkowe układu regulacji PI



Z1: f_g=const

we: T^*_{wew}, T_{zew}; wy: T_{wew}, T_{gp}, T_{gz}

Równania obiektu

$$\begin{cases} C_{vw}\dot{T}_{wew} = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ C_{vg}\dot{T}_{gp} = c_{pw}\rho_w f_g\left(T_{gz} - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \end{cases}$$

obiektS = ...
obiektG4=tf(obiektS)

Równania regulatora PI IND

s=tf('s'); reg = Kp + KTi/s

$$T_{gz} = K_p\left(T^*_{wew} - T_{wew}\right) + K_i\int\left(T^*_{wew} - T_{wew}\right)dt$$

Stan ustalony

$$\begin{cases} 0 = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ 0 = c_{pw}\rho_w f_g\left(T_{gz} - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \\ T_{wew} = T^*_{wew} \end{cases}$$

Równania stanu: ---

Funkcje: ukladGotw = reg*obiektG4(1,1)
ukladGz = feedback(ukladGotw, 1)

Bieguny (stabilność): --- pole (ukladGz)

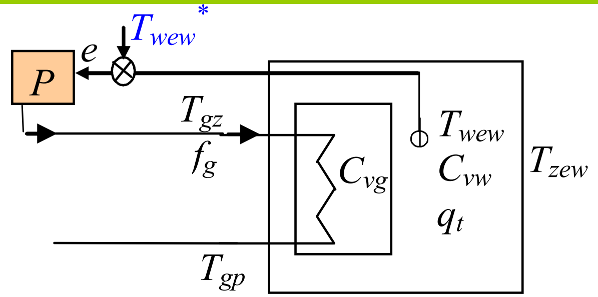
Równania statyczne: $0 = \mathbf{Ax} + \mathbf{Bu}$
lub:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -K_{cg} - K_{cw} & K_{cg} & 0 \\ K_{cg} & -c_{pw}\rho_w f_g - K_{cg} & c_{pw}\rho_w f_g \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} T_{wew} \\ T_{gp} \\ T_{gz} \end{bmatrix} + \begin{bmatrix} 0 & K_{cw} \\ 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} T^*_{wew} \\ T_{zew} \end{bmatrix}$$

Punkt równowagi: $0 = \mathbf{Ax} + \mathbf{Bu} \Rightarrow \mathbf{Ax} = -\mathbf{Bu} \Rightarrow \mathbf{x} = -\mathbf{A}^{-1}\mathbf{Bu}$

Badania podst.:
step(ukladGz)
nyquist(ukladGotw)
bode(ukladGotw)
bode(ukladGz)
pzmap(ukladGz)

1b) Pomieszczenie z grzejnikiem c.o – r.różniczkowe układu regulacji P



Z1: f_g=const

we: T^*_{wew}, T_{zew}; *wy:* T_{wew}, T_{gp}, T_{gz}

Równania obiektu

$$\begin{cases} C_{vw}\dot{T}_{wew} = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ C_{vg}\dot{T}_{gp} = c_{pw}\rho_w f_g\left(T_{gz} - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \end{cases}$$

Równania regulatora P reg = Kp

$$T_{gz} = K_p\left(T^*_{wew} - T_{wew}\right)$$

$$\begin{cases} C_{vw}\dot{T}_{wew} = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ C_{vg}\dot{T}_{gp} = c_{pw}\rho_w f_g\left(K_p(T^*_{wew} - T_{wew}) - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \end{cases}$$

Równania stanu: ukladS =ss(A,B,C,D)

$$\begin{bmatrix} \dot{T}_{wew} \\ \dot{T}_{gp} \end{bmatrix} = \begin{bmatrix} \dfrac{-K_{cg} - K_{cw}}{C_{vw}} & \dfrac{K_{cg}}{C_{vw}} \\ \dfrac{K_{cg} - c_{pw}\rho_w f_g K_p}{C_{vg}} & \dfrac{-c_{pw}\rho_w f_g - K_{cg}}{C_{vg}} \end{bmatrix} \begin{bmatrix} T_{wew} \\ T_{gp} \end{bmatrix} + \begin{bmatrix} 0 & \dfrac{K_{cw}}{C_{vw}} \\ \dfrac{c_{pw}\rho_w f_g K_p}{C_{vg}} & 0 \end{bmatrix} \begin{bmatrix} T^*_{wew} \\ T_{zew} \end{bmatrix}$$

Bieguny (stabilność): wartości własne macierzy A pole (ukladS)

Równania statyczne: $0 = \mathbf{Ax} + \mathbf{Bu}$

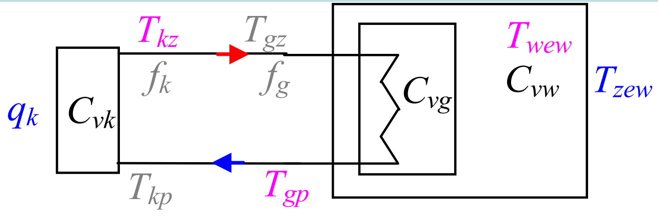
lub:

$$\begin{bmatrix} \dot{T}_{wew} \\ \dot{T}_{gp} \end{bmatrix} = \begin{bmatrix} -K_{cg} - K_{cw} & K_{cg} \\ K_{cg} - c_{pw}\rho_w f_g K_p & -c_{pw}\rho_w f_g - K_{cg} \end{bmatrix} \begin{bmatrix} T_{wew} \\ T_{gp} \end{bmatrix} + \begin{bmatrix} 0 & K_{cw} \\ c_{pw}\rho_w f_g K_p & 0 \end{bmatrix} \begin{bmatrix} T^*_{wew} \\ T_{zew} \end{bmatrix}$$

Punkt równowagi: $0 = \mathbf{Ax} + \mathbf{Bu} \Rightarrow \mathbf{Ax} = -\mathbf{Bu} \Rightarrow \mathbf{x} = -\mathbf{A}^{-1}\mathbf{Bu}$

Badania podst.:
step(ukladS)
nyquist(ukladS)
bode(ukladS)

2a) Pomieszczenie z grzejnikiem c.o + kotłownia – r.różniczkowe obiektu



$$\begin{cases} C_{vw}\dot{T}_{wew} = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ C_{vg}\dot{T}_{gp} = c_{pw}\rho_w f_g\left(T_{gz} - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \\ C_{vk}\dot{T}_{kz} = q_k - c_{pw}\rho_w f_k\left(T_{kz} - T_{kp}\right) \end{cases}$$

Z1: $f_g = f_k =$ const

we: q_k, T_{zew}; wy: T_{wew}, T_{gp}, T_{kz}

oraz $T_{gz}(t) = T_{kz}(t - T_o)$, $T_{kp}(t) = T_{gp}(t - To)$

Z2: $T_o = 0$

$$\begin{cases} C_{vw}\dot{T}_{wew} = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ C_{vg}\dot{T}_{gp} = c_{pw}\rho_w f_g\left(T_{kz} - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \\ C_{vk}\dot{T}_{kz} = q_k - c_{pw}\rho_w f_k\left(T_{kz} - T_{gp}\right) \end{cases}$$

$$c_{pw}\rho_w f_g\left(T_{kz} - T_{gp}\right) = K_{cg}\left(T_{gp} - T_{wew}\right) = K_{cw}\left(T_{wew} - T_{zew}\right) = q_k$$

Równania stanu: $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$

$$\begin{bmatrix} \dot{T}_{wew} \\ \dot{T}_{gp} \\ \dot{T}_{kz} \end{bmatrix} = [\ ] \begin{bmatrix} T_{wew} \\ T_{gz} \\ T_{kz} \end{bmatrix} + [\ ] \begin{bmatrix} q_k \\ T_{zew} \end{bmatrix}$$

Funkcje: obiektS = ss(A, B, C, D)

Bieguny (stabilność): wartości własne macierzy A

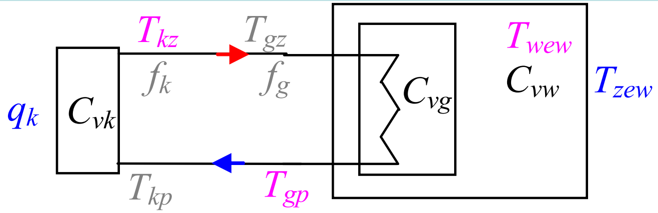
pole(obiektS)

Równania statyczne: $0 = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = [\ ] \begin{bmatrix} T_{wew} \\ T_{gz} \\ T_{kz} \end{bmatrix} + [\ ] \begin{bmatrix} q_k \\ T_{zew} \end{bmatrix}$$

Punkt równowagi: $\mathbf{x} = -\mathbf{A}^{-1}\mathbf{B}\mathbf{u}$

2a) Pomieszczenie z grzejnikiem c.o + kotłownia – transmitancje obiektu

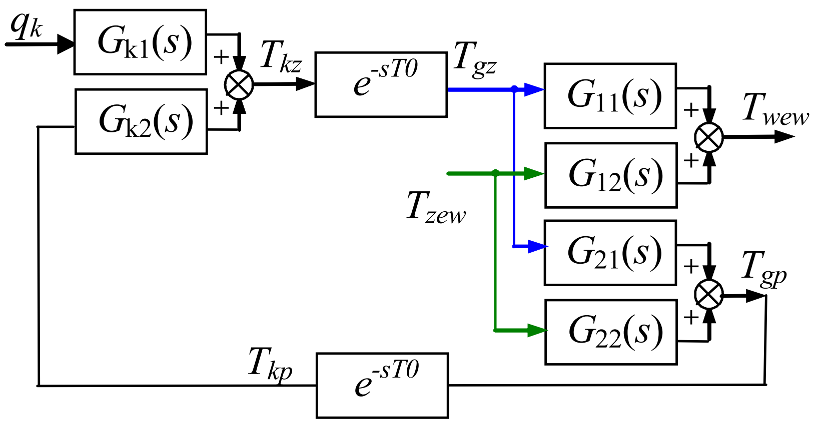


Z1: $f_g = f_k =$ const

we: q_k, T_{zew}; wy: T_{wew}, T_{gp}, T_{kz}

$$\begin{cases} C_{vw}\dot{T}_{wew} = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ C_{vg}\dot{T}_{gp} = c_{pw}\rho_w f_g\left(T_{gz} - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \\ C_{vk}\dot{T}_{kz} = q_k - c_{pw}\rho_w f_k\left(T_{kz} - T_{kp}\right) \end{cases}$$

oraz $T_{gz}(t) = T_{kz}(t - T_o)$, $T_{kp}(t) = T_{gp}(t - T_o)$



$$\begin{cases} T_{wew} = G_{11}T_{gz} + G_{12}T_{zew} \\ T_{gp} = G_{21}T_{gz} + G_{22}T_{zew} \\ T_{kz} = G_{k1}q_k + G_{k2}T_{kp} \\ T_{gz} = e^{-sT_0}T_{kz} \\ T_{kp} = e^{-sT_0}T_{gp} \end{cases}
\qquad
\begin{cases} T_{wew} = G_{11}e^{-sT_0}T_{kz} + G_{12}T_{zew} \\ T_{gp} = G_{21}e^{-sT_0}T_{kz} + G_{22}T_{zew} \\ T_{kz} = G_{k1}q_k + G_{k2}e^{-sT_0}T_{gp} \end{cases}$$

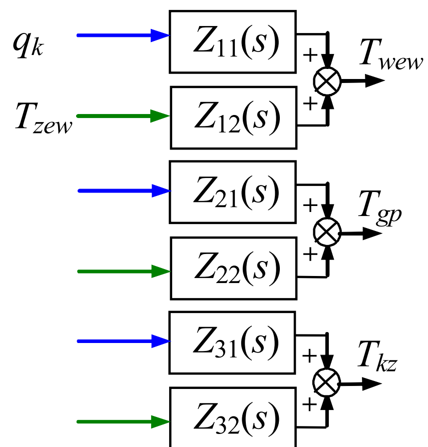
Transmitancje obiektu

$$T_{wew} = Z_{11}q_k + Z_{12}T_{zew}$$

$$T_{gp} = Z_{21}q_k + Z_{22}T_{zew}, \quad T_{kp} = e^{-sT_0}T_{gp}$$

$$T_{kz} = Z_{31}q_k + Z_{32}T_{zew}, \quad T_{gz} = e^{-sT_0}T_{kz}$$

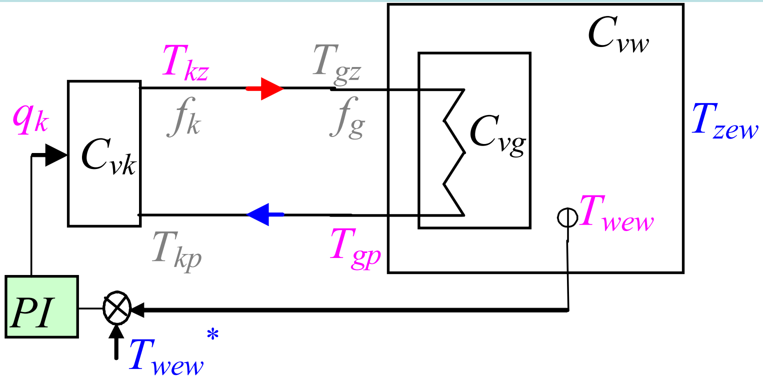
s=tf('s'); obiektZ11=...
GTo = exp(-To*s)



Bieguny (stabilność) – $M_Z(s)=0$ (dla $T_0=0$) pole(obiektZ11)

Punkt równowagi: $\lim_{t\to\infty} T_{wew}(t) = \lim_{s\to 0} sT_{wew}(s)$

2b)Pomieszczenie z grzejnikiem c.o + kotłownia – transmitancje ukł.regulacji PI



Z1: $f_g = f_k =$ const

we: T^*_{wew}, T_{zew}; wy: T_{wew}, T_{gp}, T_{gz}, q_k

Transmitancje obiektu

modelZ11= ....

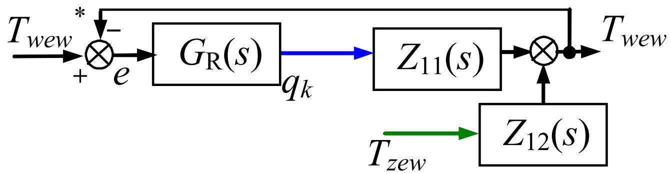
$$T_{wew} = Z_{11} q_k + Z_{12} T_{zew}$$

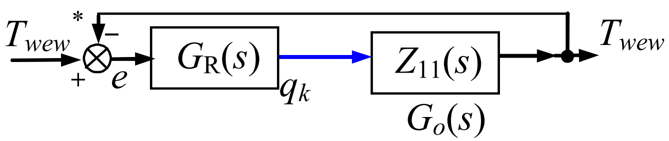
Transmitancja regulatora PI IND

s=tf('s'); reg = Kp + KTi/s

$$q_k = G_R e = \left(K_p + \frac{K_i}{s} \right) \left(T^*_{wew} - T_{wew} \right)$$

Transmitancje układu





$$G_e = \frac{e}{T^*_{wew}} = \frac{1}{1 + G_R G_o}$$

$$G_z = \frac{T_{wew}}{T^*_{wew}} = \frac{G_R G_o}{1 + G_R G_o}$$

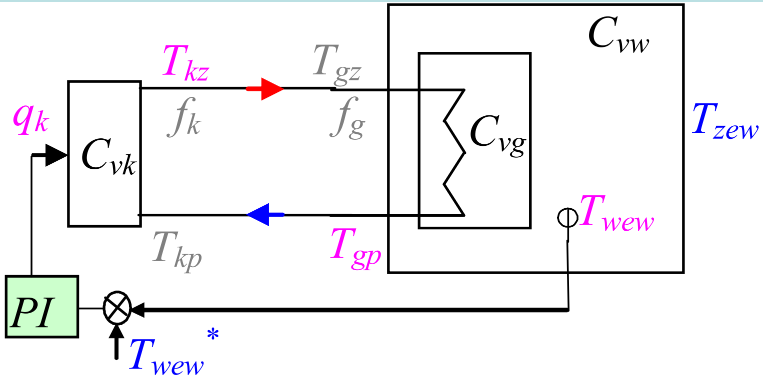
ukladGotw = reg*modelZ11
ukladGz = feedback(ukladGotw, 1)

Badania podst.:
step(ukladGz)
nyquist(ukladGotw)
bode(ukladGotw)
bode(ukladGz)
pzmap(ukladGz)

Bieguny (stabilność) – $M_{Gz}(s)=0$ (dla $T_0=0$) pole(ukladGz)

Punkt równowagi: $T_{wew} = T^*_{wew}$

2b)Pomieszczenie z grzejnikiem c.o + kotłownia – r.różniczkowe ukł.regulacji PI



Z1: $f_g = f_k = \text{const}$

we: T^*_{wew}, T_{zew}; wy: T_{wew}, T_{gp}, T_{gz}, q_k

Równania obiektu Z2: $T_o = 0$

obiektS = ...
obiektG4=tf(obiektS)

$$\begin{cases} C_{vw}\dot{T}_{wew} = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ C_{vg}\dot{T}_{gp} = c_{pw}\rho_w f_g\left(T_{kz} - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \\ C_{vk}\dot{T}_{kz} = q_k - c_{pw}\rho_w f_k\left(T_{kz} - T_{gp}\right) \end{cases}$$

Równania regulatora PI IND

s=tf('s'); reg = Kp + KTi/s

$$q_k = K_p\left(T^*_{wew} - T_{wew}\right) + K_i\int\left(T^*_{wew} - T_{wew}\right)dt$$

Stan ustalony

$$\begin{cases} 0 = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ 0 = c_{pw}\rho_w f_g\left(T_{kz} - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \\ 0 = q_k - c_{pw}\rho_w f_k\left(T_{kz} - T_{gp}\right) \\ T_{wew} = T^*_{wew} \end{cases}$$

Równania stanu: ---

Funkcje: ukladGotw = reg*obiektZ11
ukladGz = feedback(ukladGotw, 1)

Bieguny (stabilność): --- pole (ukladGz)

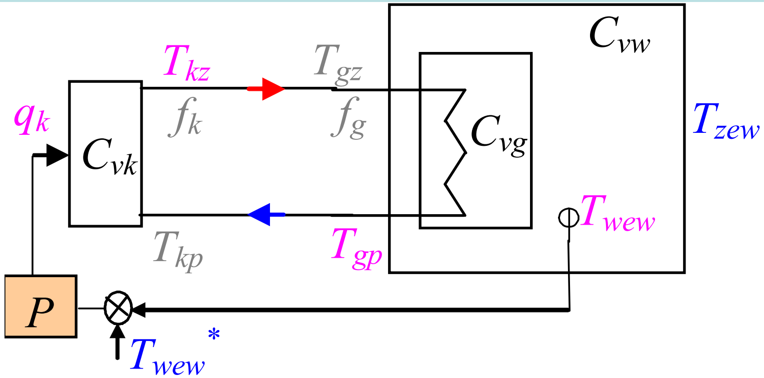
Równania statyczne: $0 = \mathbf{Ax} + \mathbf{Bu}$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -K_{cg} - K_{cw} & K_{cg} & 0 & 0 \\ K_{cg} & -c_{pw}\rho_w f_g - K_{cg} & c_{pw}\rho_w f_g & 0 \\ 0 & c_{pw}\rho_w f_g & -c_{pw}\rho_w f_g & 1 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_{wew} \\ T_{gp} \\ T_{kz} \\ q_k \end{bmatrix} + \begin{bmatrix} 0 & K_{cw} \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} T^*_{wew} \\ T_{zew} \end{bmatrix}$$

Punkt równowagi: $0 = \mathbf{Ax} + \mathbf{Bu} \Rightarrow \mathbf{Ax} = -\mathbf{Bu} \Rightarrow \mathbf{x} = -\mathbf{A}^{-1}\mathbf{Bu}$

Badania podst.:
step(ukladGz)
nyquist(ukladGz)
bode(ukladGz)

2b) Pomieszczenie z grzejnikiem c.o + kotłownia – r.różniczkowe ukł.regulacji P



Z1: $f_g = f_k = \text{const}$

we: T^*_{wew}, T_{zew}; wy: T_{wew}, T_{gp}, T_{gz}, q_k

Równania obiektu Z2: $T_o = 0$

$$\begin{cases} C_{vw}\dot{T}_{wew} = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ C_{vg}\dot{T}_{gp} = c_{pw}\rho_w f_g\left(T_{kz} - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \\ C_{vk}\dot{T}_{kz} = q_k - c_{pw}\rho_w f_k\left(T_{kz} - T_{gp}\right) \end{cases}$$

Równania regulatora P reg = Kp

$$q_k = K_p\left(T^*_{wew} - T_{wew}\right)$$

$$\begin{cases} C_{vw}\dot{T}_{wew} = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ C_{vg}\dot{T}_{gp} = c_{pw}\rho_w f_g\left(T_{kz} - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \\ C_{vk}\dot{T}_{kz} = K_p\left(T^*_{wew} - T_{wew}\right) - c_{pw}\rho_w f_k\left(T_{kz} - T_{gp}\right) \end{cases}$$

ukladS =ss(A,B,C,D)

Równania stanu: $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$

$$\begin{bmatrix} \dot{T}_{wew} \\ \dot{T}_{gp} \\ \dot{T}_{kz} \end{bmatrix} = \left[\quad\right] \begin{bmatrix} T_{wew} \\ T_{gz} \\ T_{kz} \end{bmatrix} + \left[\quad\right] \begin{bmatrix} q_k \\ T_{zew} \end{bmatrix}$$

Bieguny (stabilność): wartości własne macierzy A pole (ukladS)

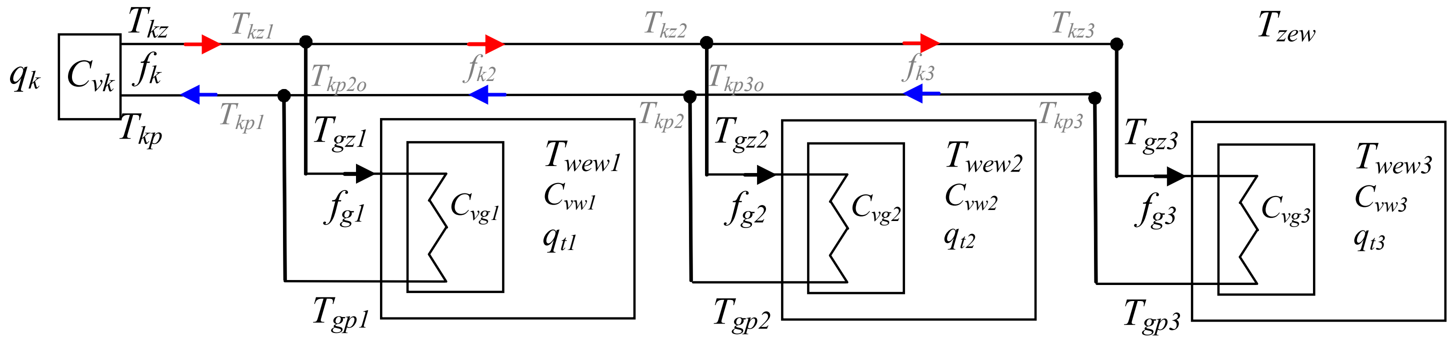
Równania statyczne: $0 = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$
lub:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \left[\quad\right] \begin{bmatrix} T_{wew} \\ T_{gz} \\ T_{kz} \end{bmatrix} + \left[\quad\right] \begin{bmatrix} q_k \\ T_{zew} \end{bmatrix}$$

Punkt równowagi: $0 = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \Rightarrow \mathbf{A}\mathbf{x} = -\mathbf{B}\mathbf{u} \Rightarrow \mathbf{x} = -\mathbf{A}^{-1}\mathbf{B}\mathbf{u}$

Badania podst.:
step(ukladS)
nyquist(ukladS)
bode(ukladS)

3a) 3*pomieszczenie z grzejnikiem c.o + kotłownia – r.różniczkowe obiektu



Z1: f_{gi}=const, f_k= const

we: q_k, T_{zew}; wy: T_{wewi}, T_{gpi}, T_{kz}

$$\begin{cases} C_{vw1}\dot{T}_{wew1} = K_{cg1}\left(T_{gp1} - T_{wew1}\right) - K_{cw1}\left(T_{wew1} - T_{zew1}\right) \\ C_{vg1}\dot{T}_{gp1} = c_{pw}\rho_w f_{g1}\left(T_{gz1} - T_{gp1}\right) - K_{cg1}\left(T_{gp1} - T_{wew1}\right) \\ \dots\dots\dots\dots\dots\dots \\ C_{vk}\dot{T}_{kz} = q_k - c_{pw}\rho_w f_k\left(T_{kz} - T_{kp}\right) \end{cases}$$

$$f_{k3} = f_{g3} \qquad f_{k2} = f_{g2} + f_{k3} \qquad f_k = f_{g1} + f_{k2}$$

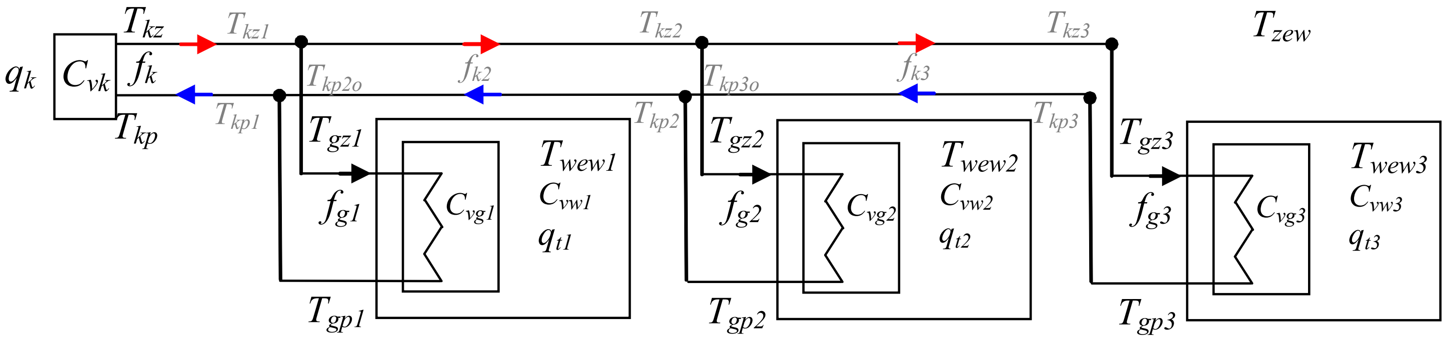
$$T_{kz1}(t) = T_{kz}(t - T_{o1}) \qquad T_{kp1}(t) = \frac{T_{gp1}(t) f_{g1}(t) + T_{kp2o}(t) f_{k2}(t)}{f_{g1}(t) + f_{k2}(t)} \qquad T_{kp}(t) = T_{kp1}(t - T_{o1})$$

$$T_{kz2}(t) = T_{kz1}(t - T_{o2}) \qquad T_{kp2}(t) = \frac{T_{gp2}(t) f_{g2}(t) + T_{kp3o}(t) f_{k3}(t)}{f_{g2}(t) + f_{k3}(t)} \qquad T_{kp2o}(t) = T_{kp2}(t - T_{o2})$$

$$T_{kp3o}(t) = T_{kp3}(t - T_{o3})$$

$$T_{kz3}(t) = T_{kz2}(t - T_{o3}) \qquad T_{kp3}(t) = T_{gp3}(t)$$

3a) 3*pomieszczenie z grzejnikiem c.o + kotłownia – r.różniczkowe obiektu



Z1: f_{gi} =const, f_k= const

we: q_k, T_{zew}; wy: T_{wewi}, T_{gpi}, T_{kz}

Z2: $T_{o1} = T_{o2} = T_{o3} = 0$

$$\begin{cases} C_{vw1}\dot{T}_{wew1} = K_{cg1}\left(T_{gp1} - T_{wew1}\right) - K_{cw1}\left(T_{wew1} - T_{zew1}\right) \\ C_{vg1}\dot{T}_{gp1} = c_{pw}\rho_w f_{g1}\left(T_{gz1} - T_{gp1}\right) - K_{cg1}\left(T_{gp1} - T_{wew1}\right) \\ \dots\dots\dots\dots\dots\dots\dots \\ C_{vk}\dot{T}_{kz} = q_k - c_{pw}\rho_w f_k\left(T_{kz} - T_{kp}\right) \end{cases}$$

Równania stanu: $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$

$$\begin{bmatrix} \dot{T}_{wew1} \\ \dot{T}_{gp1} \\ \dot{T}_{wew2} \\ \dot{T}_{gp2} \\ \dot{T}_{wew3} \\ \dot{T}_{gp3} \\ \dot{T}_{kz} \end{bmatrix} = \begin{bmatrix} \quad & & & & & & \quad \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} T_{wew1} \\ T_{gp1} \\ T_{wew2} \\ T_{gp2} \\ T_{wew3} \\ T_{gp3} \\ T_{kz} \end{bmatrix} + \begin{bmatrix} \quad & \quad \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} q_k \\ T_{zew} \end{bmatrix}$$

Funkcje: obiektS = ss(A, B, C, D)

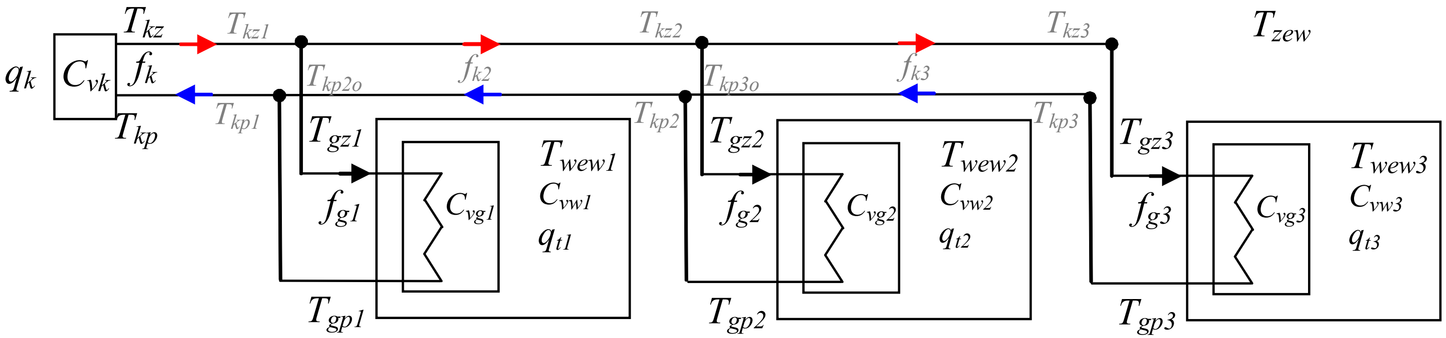
Bieguny (stabilność): wartości własne macierzy A

pole(obiektS)

Równania statyczne: $0 = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$

Punkt równowagi: $\mathbf{x} = -\mathbf{A}^{-1}\mathbf{B}\mathbf{u}$

3a) 3*pomieszczenie z grzejnikiem c.o + kotłownia – transmitancje obiektu



Z1: f_{gi}=const, f_k= const

we: q_k, T_{zew}; wy: T_{wewi}, T_{gpi}, T_{kz}

$$\begin{cases} T_{wewi} = G_{11i}T_{gzi} + G_{12i}T_{zew} \\ T_{gpi} = G_{21i}T_{gzi} + G_{22i}T_{zew} \\ \dots\dots\dots\dots \\ T_{kz} = G_{k1}q_k + G_{k2}T_{kp} \end{cases}$$

oraz

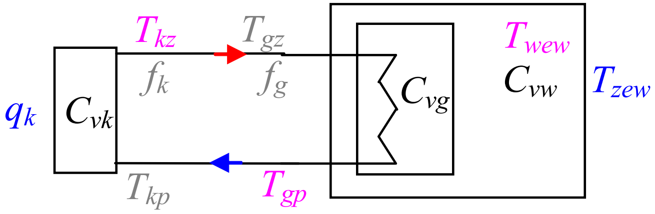
$$T_{kz1} = e^{-sT_{o1}}T_{kz}$$
$$T_{kp1} = k_{1a}T_{gp1} + k_{1b}T_{kp2o}$$
$$T_{kp} = e^{-sT_{o1}}T_{kp1}$$
$$T_{kz2} = e^{-sT_{o2}}T_{kz1}$$
$$T_{kp2} = k_{2a}T_{gp2} + k_{2b}T_{kp3o}$$
$$T_{kp2o} = e^{-sT_{o2}}T_{kp2}$$
$$T_{kz3} = e^{-sT_{o3}}T_{kz2}$$
$$T_{kp3} = T_{gp3}$$
$$T_{kp3o} = e^{-sT_{o3}}T_{kp3}$$

Transmitancje układu:

$$T_{wewi} = Z_{11i}q_k + Z_{12i}T_{zew}$$
$$\dots\dots\dots$$
$$T_{gp} = Z_{21}q_k + Z_{22}T_{zew}$$
$$T_{kz} = Z_{31}q_k + Z_{32}T_{zew}$$

4) Opóźnienia transportowe

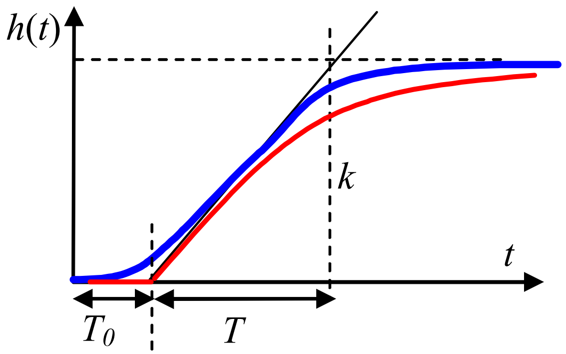
Opóźnienia transportowe opisujące zjawiska



$$\begin{cases} C_{vw}\dot{T}_{wew} = K_{cg}\left(T_{gp} - T_{wew}\right) - K_{cw}\left(T_{wew} - T_{zew}\right) \\ C_{vg}\dot{T}_{gp} = c_{pw}\rho_w f_g\left(T_{gz} - T_{gp}\right) - K_{cg}\left(T_{gp} - T_{wew}\right) \\ C_{vk}\dot{T}_{kz} = q_k - c_{pw}\rho_w f_k\left(T_{kz} - T_{kp}\right) \end{cases}$$

oraz $T_{gz}(t)=T_{kz}(t-T_o)$, $T_{kp}(t)=T_{gp}(t-To)$

Opóźnienia transportowe wynikające z identyfikacji, np.:



$$\frac{k}{Ts+1}e^{-sT_o}$$

Funkcje: s=tf(‘s’)
opoz = exp(-s*To)

Go1 = k / (T*s+1) *exp(-s*To)

Go1 = tf(k, [T, 1], ‘ioDelay’, To)
Go1 = tf(k, [T, 1], 'InputDelay', To)
Go1 = tf(k, [T, 1], 'OutputDelay', To)

---

aproksymacja Padé $$e^{-sT_0} \approx \frac{1 - sT_0/2}{1 + sT_0/2}$$

opozPade = pade(opoz,1)

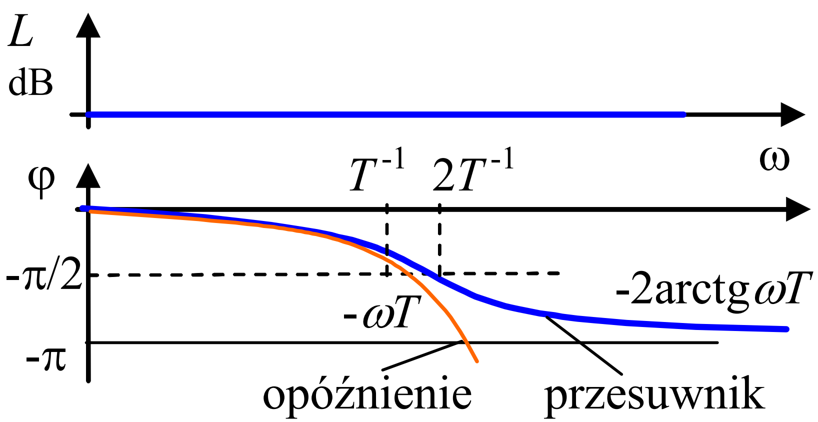
4) Opóźnienia transportowe - aproksymacja Padé

$$e^{-sT_o} = \frac{e^{-sT_o/2}}{e^{sT_o/2}} = \frac{1 + \frac{\left(-\frac{sT_o}{2}\right)^i}{i!} + \ldots}{1 + \frac{\left(\frac{sT_o}{2}\right)^i}{i!} + \ldots} = \frac{1 - \frac{sT_o}{2} + \frac{s^2 T_o^2}{8} + \ldots}{1 + \frac{sT_o}{2} + \frac{s^2 T_o^2}{8} + \ldots} \approx \frac{1 - s\frac{T_0}{2}}{1 + s\frac{T_0}{2}}$$

opóźnienie	≈	przesuwnik fazowy

$$G(j\omega) = e^{-j\omega T_o}$$

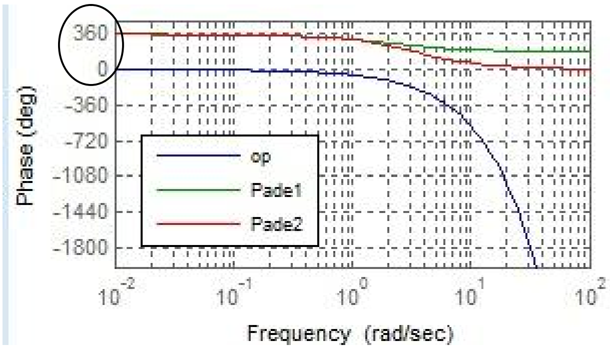
$$G(j\omega) = \frac{1 - j\omega T}{1 + j\omega T} = \frac{\left(1 + \omega^2 T^2\right) e^{j\mathrm{arctg}(-\omega T)}}{\left(1 + \omega^2 T^2\right) e^{j\mathrm{arctg}(\omega T)}} = e^{-j2\mathrm{arctg}(\omega T)}$$

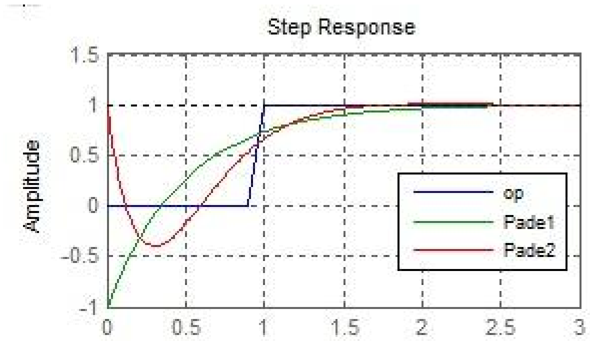


4) Opóźnienia transportowe - aproksymacja Padé

$$e^{-sT_o} \approx \frac{1 - sT_o/2}{1 + sT_o/2}$$

$$e^{-s} \approx \frac{1 - 0.5s}{1 + 0.5s}$$



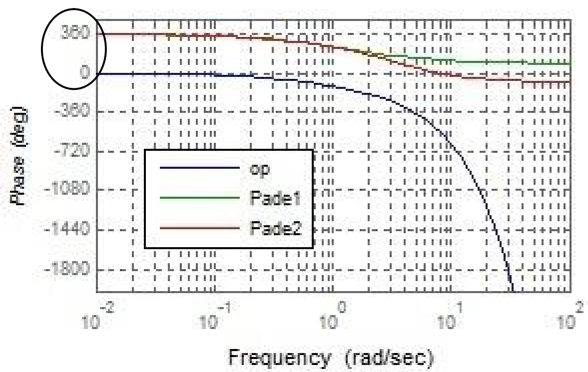


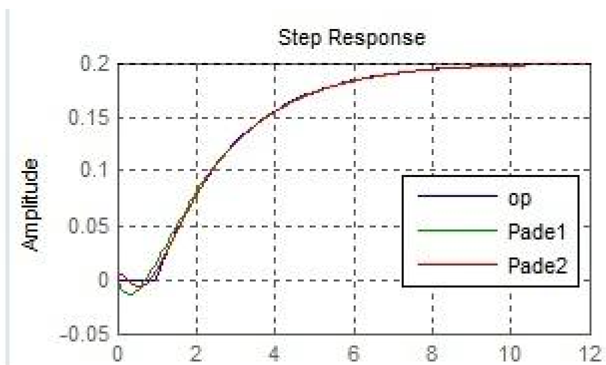

```
To=1; s=tf('s');
op = exp(-s*To);
Pade1 = pade(op, 1);
Pade2 = pade(op, 2);
```

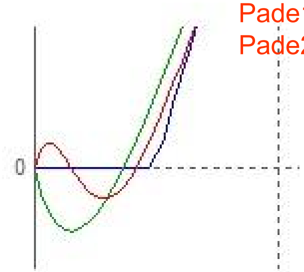
$$\frac{k}{Ts+1} e^{-sT_o} \approx \frac{k}{Ts+1} \cdot \frac{1 - sT_o/2}{1 + sT_o/2}$$

$$\frac{0.2}{2s+1} e^{-s} \approx \frac{0.2}{2s+1} \cdot \frac{1 - 0.5s}{1 + 0.5s}$$

```
k=0.2; T=2; To=1;
s=tf('s');
op = k/(T*s+1)*exp(-s*To);
Pade1 = pade(op, 1);
Pade2 = pade(op, 2);
```







Models with Time Delays:

- First Order Plus Dead Time Model
- Input and Output Delay in State-Space Model
- Transport Delay in MIMO Transfer Function
- Closing Feedback Loops with Time Delays
- Time-Delay Approximation in Continuous-Time Open-Loop Model
- Time-Delay Approximation in Continuous-Time Closed-Loop Model

http://www.mathworks.com/help/control/models-with-time-delays.html