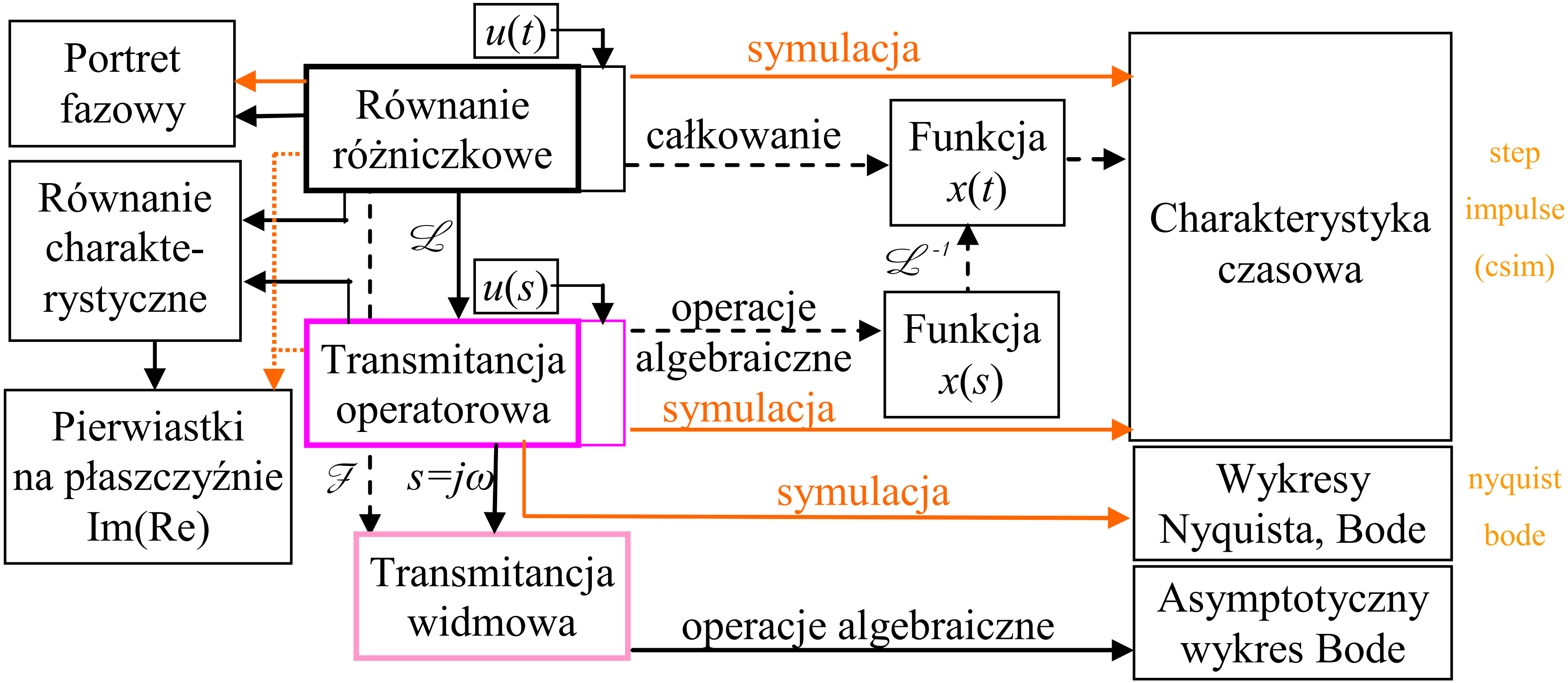
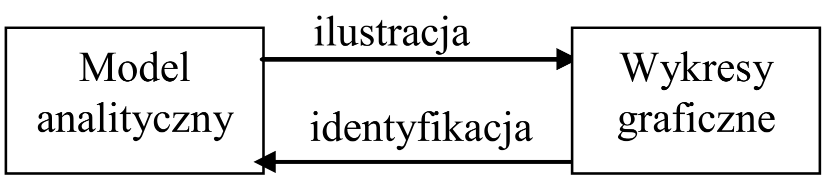
Opis analityczny i graficzny





Opis dynamiki obiektu

Równanie n-tego rzędu

$$a_n x^{(n)}(t) + \ldots + a_1 \dot{x}(t) + a_0 x(t) = u(t)$$

$$a_n \lambda^n + \ldots + a_1 \lambda + a_0 = 0$$

$$a_n (\lambda - \lambda_k) \ldots . (\lambda^2 + b\lambda + c) = 0$$

Równania stanu

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

Transmitancje

$$\frac{X(s)}{U(s)} = G(s) = \frac{L(s)}{M(s)}$$

Odpowiedź skokowa / implusowa

odpowiedź na skokową zmianę wymuszenia (dowolny skok)

Położenie biegunów i zer

Charakterystyki częstotliwościowe:

$$G(j\omega) = \frac{L(j\omega)}{M(j\omega)} = P(\omega) + jQ(\omega) = M(\omega)e^{j\varphi(\omega)}$$

ch-ka rzeczywista	- P(ω) = Re(G(jω))
ch-ka urojona	- Q(ω) = Im(G(jω))
ch. aplitudowo-fazowa	- Q(P) (ch.Nyquista - dla ukł. otwartych)
ch. amplitudowa	- M(ω) = |G(jω)|
ch. fazowa	- φ(ω)
logarytmiczna ch. modułu	- L(ω)= 20 lgM(ω)= 20 lg|G(jω)|
logarytmiczna ch. fazy	- φ(ω) = arctg(Q/P)
log.ch.amplitudowo-fazowa	- L(φ)

Opis dynamiki obiektu

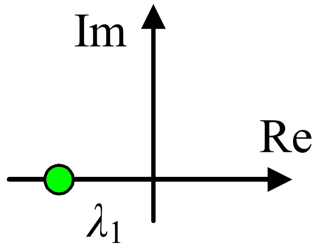
Równanie 1. rzędu

$$b\dot{x}(t) + cx(t) = u(t)$$

$$b\lambda + c = 0$$

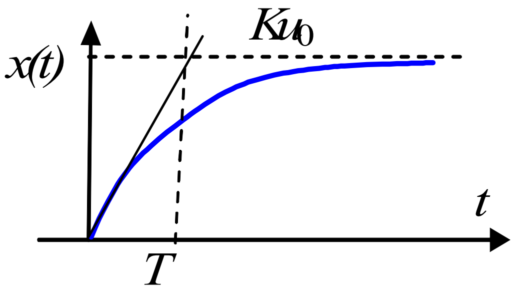
$$\lambda_1 = \frac{-c}{b}$$

$$c > 0, b > 0$$



$$G(s) = \frac{K}{Ts + 1}$$

człon inercyjny



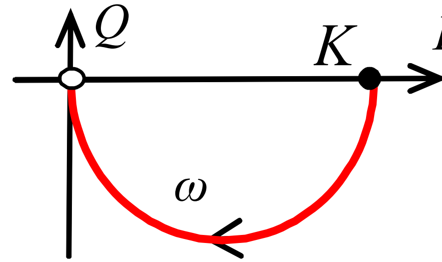
$$u(t) = 1(t) \qquad x(0) = 0$$

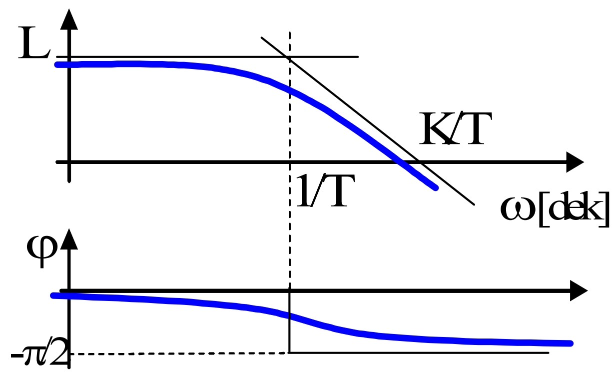
$$x(t) = -\frac{1}{c}e^{-(c/b)t} + \frac{1}{c}$$

$$G(j\omega) = \frac{K}{1 + j\omega T} = \frac{K}{1 + \omega^2 T^2} - j\frac{K\omega T}{1 + \omega^2 T^2}$$

$$P(\omega) = \frac{K}{1 + \omega^2 T^2}$$

$$Q(\omega) = -\frac{K\omega T}{1 + \omega^2 T^2}$$





$$L(\omega) = 20\lg M(\omega) = 20\lg|G(j\omega)| = 20\lg\left|\frac{K}{1 + j\omega T}\right|$$

$$\varphi(\omega) = arctg(Q/P) = arctg(-\omega T)$$

Opis dynamiki obiektu

Równanie 2. rzędu

$$a_2\ddot{x}(t)+a_1\dot{x}(t)+a_0x(t)=b_0u(t)$$

$$\ddot{x}(t)+b\dot{x}(t)+cx(t)=u(t)$$

$$\ddot{x}(t)+2\xi\,\omega_n\,\dot{x}(t)+\omega_n^2x(t)=u(t) \qquad \omega_n>0$$

$$\lambda^2+2\xi\omega_n\lambda+\omega_n^2=0$$

człon oscylacyjny

$$\ddot{x}(t)+2\xi\,\omega_n\,\dot{x}(t)-\omega_n^2x(t)=u(t) \qquad \omega_n>0$$

$$\lambda^2+2\xi\omega_n\lambda-\omega_n^2=0$$

Człon oscylacyjny znormalizowany

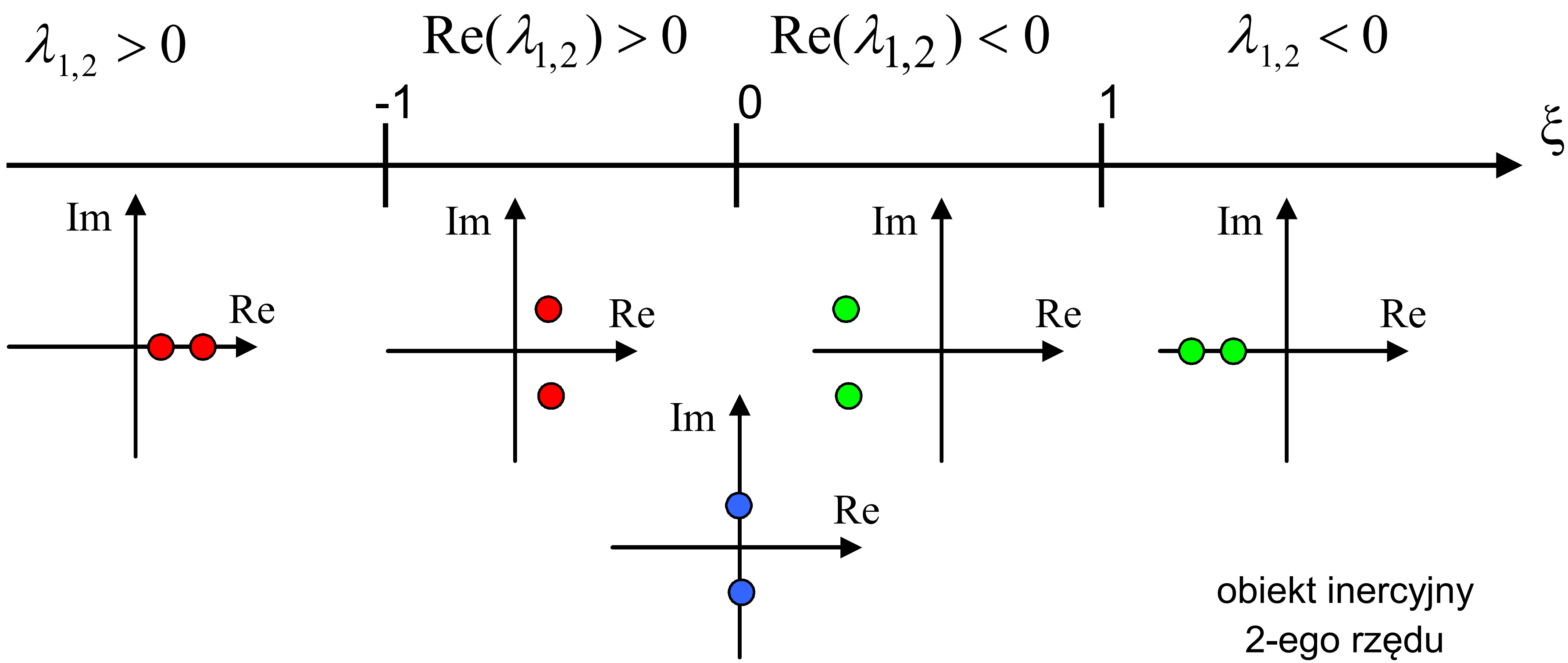
$\dfrac{\omega_n^2}{s^2+2\,\xi\,\omega_n s+\omega_n^2},\ \omega_n>0$	$\dfrac{1}{T_n^2s^2+2\,\xi\,T_n\,s+1},\ T_n>0$	$\dfrac{\sigma^2+\omega_r^2}{s^2+2\,\sigma s+\sigma^2+\omega_r^2}$
$s_1=-\xi\omega_n+\omega_n\sqrt{\xi^2-1}$ $s_2=-\xi\omega_n-\omega_n\sqrt{\xi^2-1}$	Gdy $\xi^2-1<0$: $s_1=\alpha+j\omega_r$, $s_2=\alpha-j\omega_r$ gdzie: $\alpha=-\xi\omega_n$, $\omega_r=\omega_n\sqrt{1-\xi^2}$	$s_1=-\sigma+j\omega_r$ $s_2=-\sigma-j\omega_r$ gdzie: $\sigma=\xi\omega_n$, $\omega_r=\omega_n\sqrt{1-\xi^2}$

$$\frac{\sigma^2+\omega_r^2}{s^2+2\,\sigma s+\sigma^2+\omega_r^2}=\frac{(\xi\omega_n)^2+\left(\omega_n\sqrt{1-\xi^2}\right)^2}{s^2+2\,\xi\omega_n s+(\xi\omega_n)^2+\left(\omega_n\sqrt{1-\xi^2}\right)^2}=\frac{\omega_n^2}{s^2+2\,\xi\omega_n s+\omega_n^2}$$

Człon oscylacyjny (równanie oscylacyjne)

$$\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0 \qquad \omega_n > 0$$

$$\lambda_{1,2} = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2} \longrightarrow \lambda_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$



obiekt inercyjny
2-ego rzędu

Położenie biegunów a odpowiedź skokowa

$$\ddot{x}(t) + 2\xi\,\omega_n\,\dot{x}(t) + \omega_n^2 x(t) = \omega_n^2 u(t) \quad , \xi \geq 0$$

$$\lambda_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$$\lambda_{1,2} = \alpha \pm j\omega_r$$

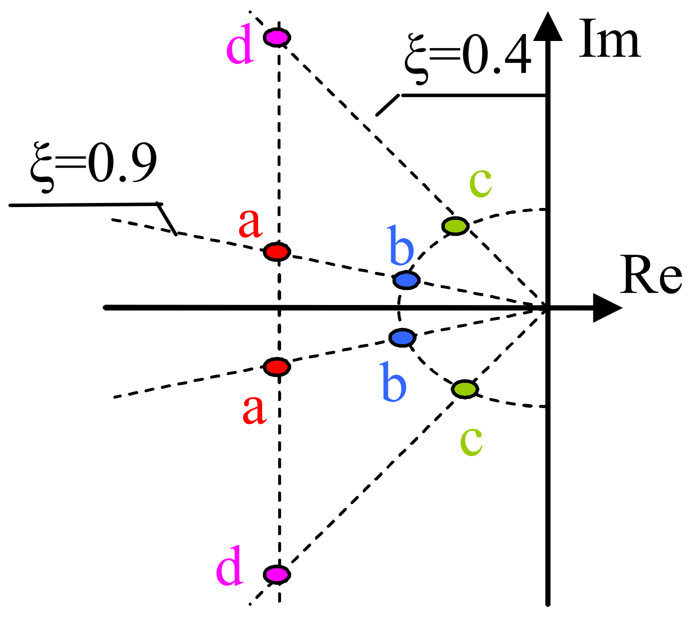
$$\alpha = -\xi\omega_n$$

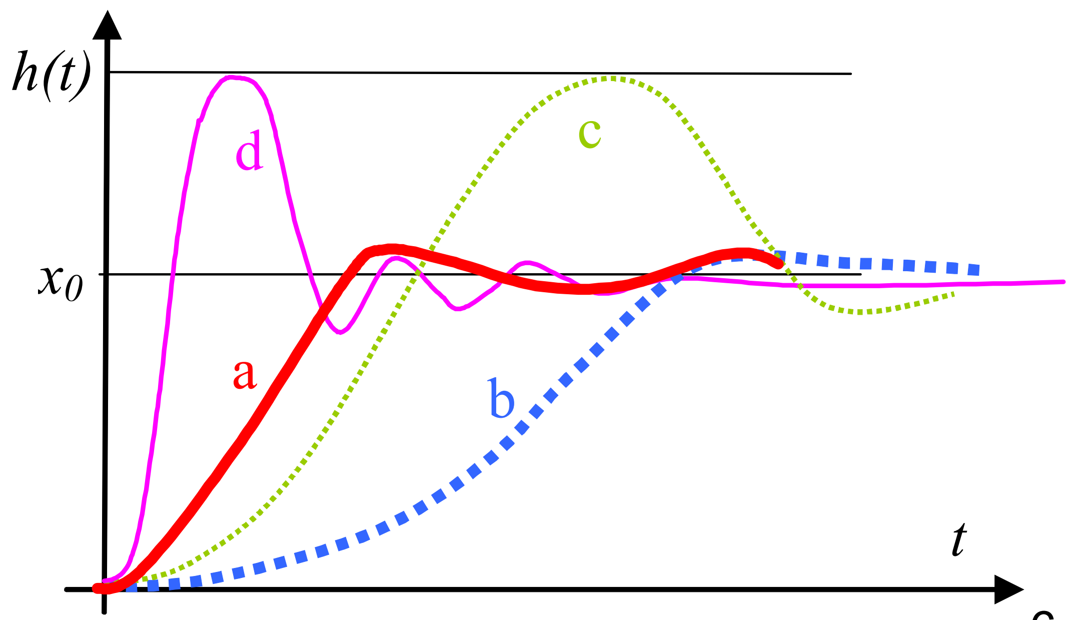
$$\omega_r = \omega_n\sqrt{1-\xi^2}$$

$$u(t) = 1(t) \qquad x(0) = 0 \qquad \dot{x}(0) = 0$$

$$x(t) = 1 - e^{\alpha t}\left(\cos\omega_r t - \frac{\alpha}{\omega_r}\sin\omega_r t\right) =$$

$$= 1 - Ae^{\alpha t}\sin(\omega_r t + \varphi)$$





Położenie biegunów a parametry równania

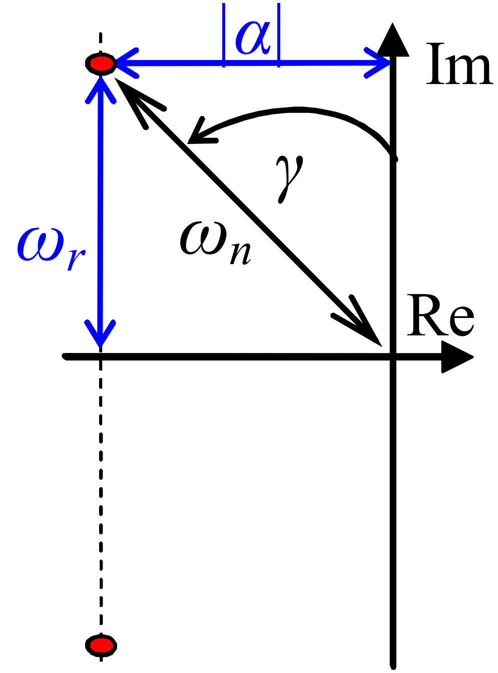
$$\ddot{x}(t) + 2\xi\,\omega_n\,\dot{x}(t) + \omega_n^2 x(t) = \omega_n^2 u(t) \quad ,\xi \geq 0$$

$$\lambda_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$$\lambda_{1,2} = \alpha \pm j\omega_r$$

$$\alpha = -\xi\omega_n$$

$$\omega_r = \omega_n\sqrt{1-\xi^2}$$



$$\sqrt{\alpha^2 + \omega_r^2} = \sqrt{(-\xi\omega_n)^2 + \left(\omega_n\sqrt{1-\xi^2}\right)^2} = \omega_n$$

$$\sin\gamma = \frac{|\alpha|}{\sqrt{\alpha^2 + \omega_r^2}} = \frac{\xi\omega_n}{\sqrt{\xi^2\omega_n^2 + \omega_n^2(1-\xi^2)}} = \xi$$

Równanie oscylacyjne a bezpośrednie wskaźniki jakości

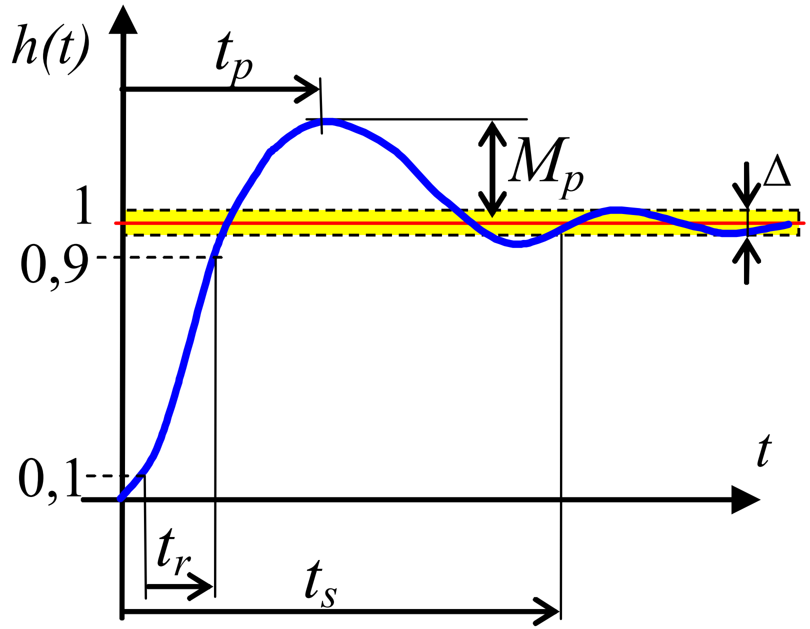
$$\ddot{x}(t) + 2\xi\omega_n \dot{x}(t) + \omega_n^2 x(t) = \omega_n^2 u(t)$$

$$x(t) = 1 - e^{\alpha t}\left(\cos\omega_r t - \frac{\alpha}{\omega_r}\sin\omega_r t\right) =$$

$$= 1 - Ae^{\alpha t}\sin(\omega_r t + \varphi)$$

$$s_{1,2} = \alpha \pm j\omega_r$$

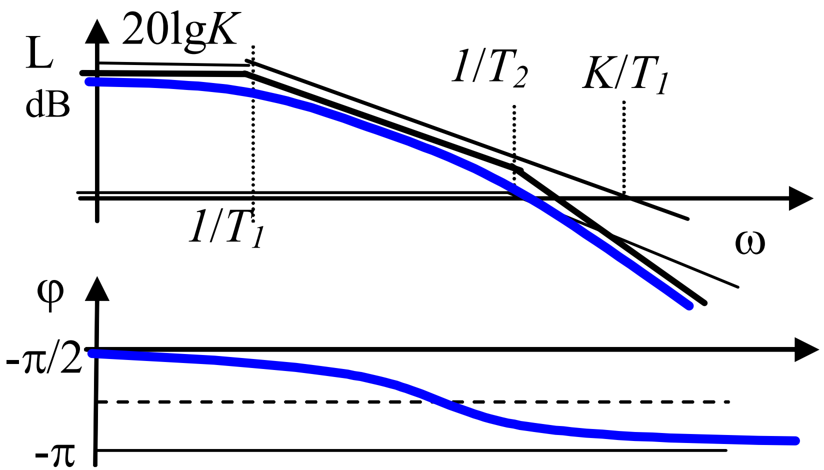
$$0 < \xi < 1 \quad \leftrightarrow \quad \alpha < 0$$



Charakterystyki częstotliwościowe

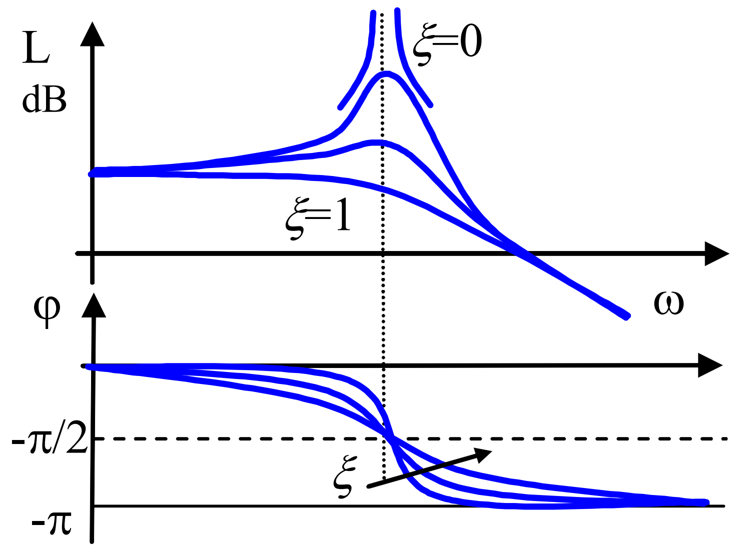
$$G(s) = \frac{K}{(T_1 s + 1)(sT_2 + 1)}$$

$$G(j\omega) = \frac{K}{(j\omega T_1 + 1)(j\omega T_2 + 1)}$$

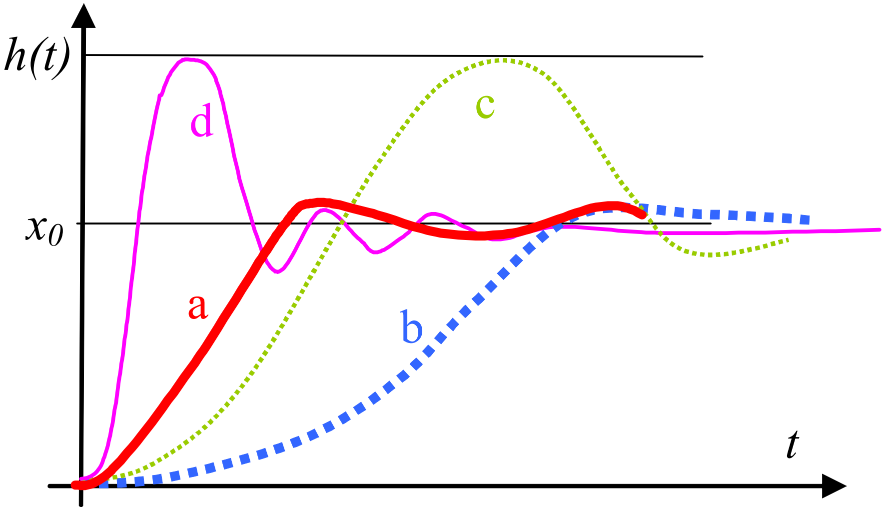


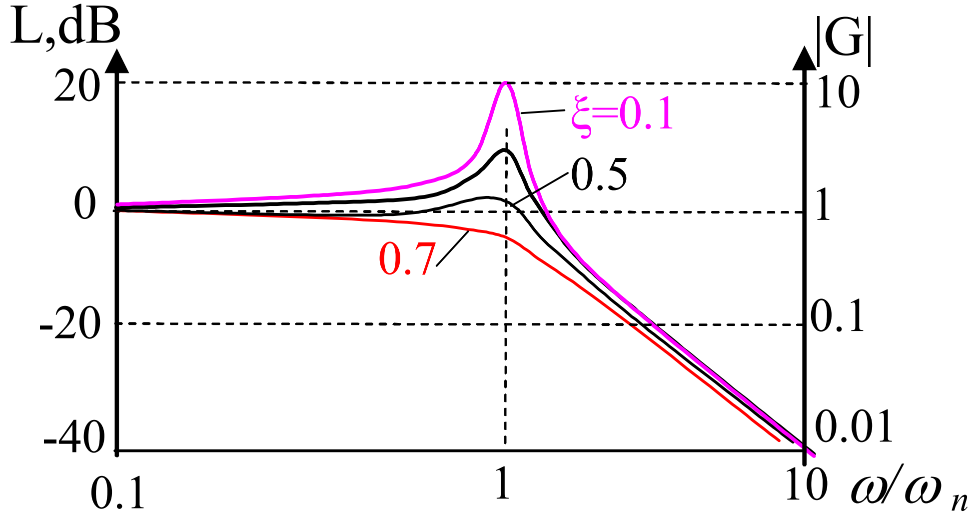
$$G(s) = \frac{K}{T_n^2 s^2 + 2\xi T_n s + 1}$$

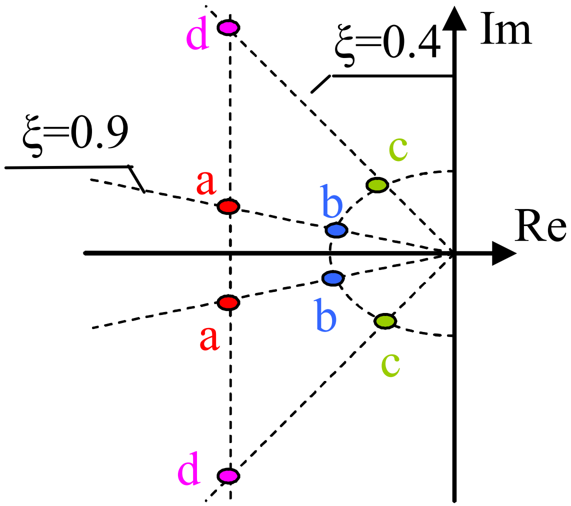
$$G(j\omega) = \frac{K}{(j\omega T_n)^2 + j2\xi T_n \omega + 1}$$

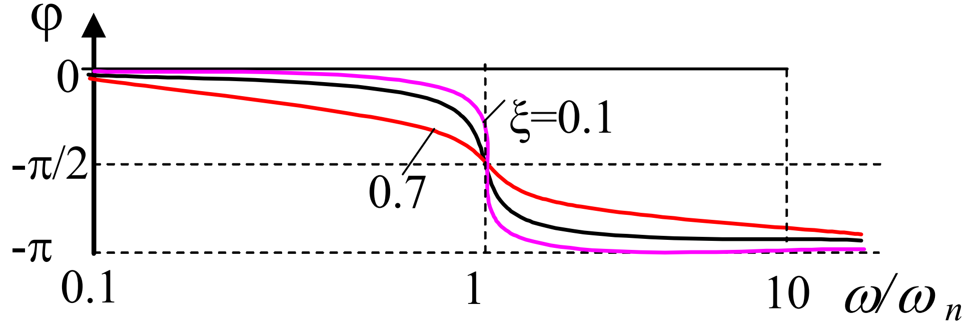


Analiza liniowa







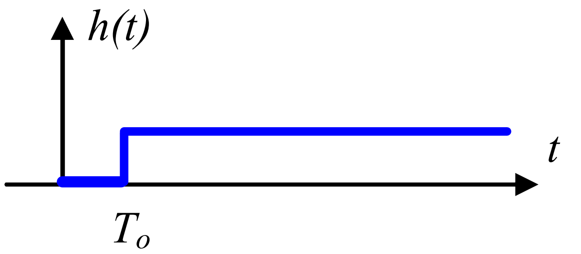


$$\ddot{x}(t) + 2\xi\, \omega_n\, \dot{x}(t) + \omega_n^2 x(t) = \omega_n^2 u(t)$$

Człon opóźniający ≈ przesuwnik fazowy

$$x(t) = u(t - T_o)$$

$$h(t) = 1(t - T_o)$$



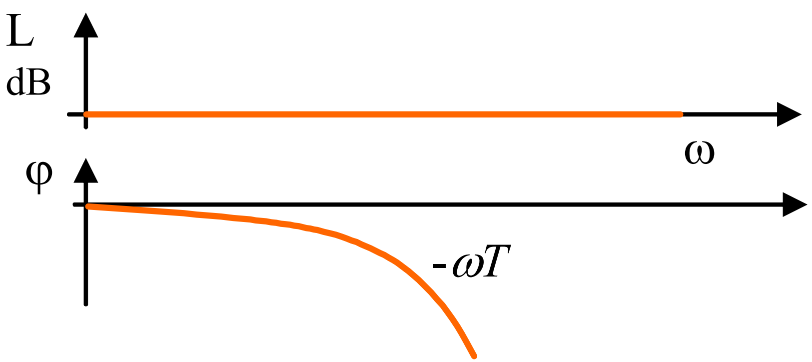
$$G(s) = e^{-sT_o} = \frac{e^{-s\frac{T_0}{2}}}{e^{s\frac{T_0}{2}}} = \frac{1 - s\frac{T_0}{2} + \ldots}{1 + s\frac{T_0}{2} + \ldots} \approx \frac{1 - s\frac{T_0}{2}}{1 + s\frac{T_0}{2}}$$

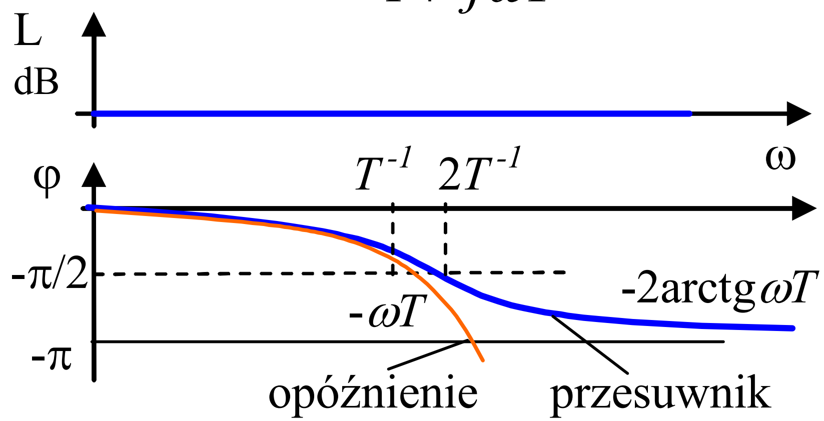
aproksymacja Padé 1-ego rzędu

$$G(s) = e^{-sT_o}$$

$$G(j\omega) = e^{-j\omega T_o}$$

$$G(j\omega) = \frac{1 - j\omega T}{1 + j\omega T}$$





Przykład

$$G_1 = \frac{2}{2s+1}, G_2 = \frac{2}{20s+1}, \; G_3 = \frac{10}{2s+1}$$

```
G1 = tf(2, [2 ,1]);     %1-niebieskie kropki
G2 = tf(2, [20,1]);     %2-czerwona linia
G3 = tf(4, [2, 1]);     %3-zielone kreski
```

```
s=tf('s');
G1 = 2/(2*s+1);     %1-niebieskie kropki
G2= 2/(20*s+1);     %2-czerwona linia
G3 = 4/(2*s+1);     %3-zielone kreski
```

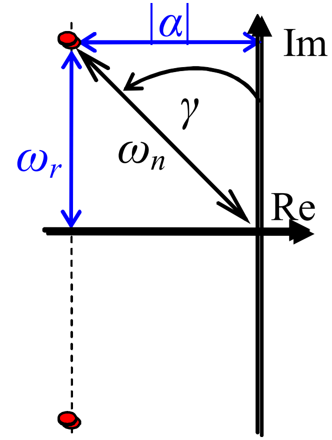
```
figure; step(G1,'b.', G2,'r', G3, 'g--');
figure; pzmap(G1,'b.', G2,'r', G3, 'g--');
figure; nyquist(G1,'b.', G2,'r', G3, 'g--');
figure; bode(G1,'b.', G2,'r', G3, 'g--');
```

Przykład

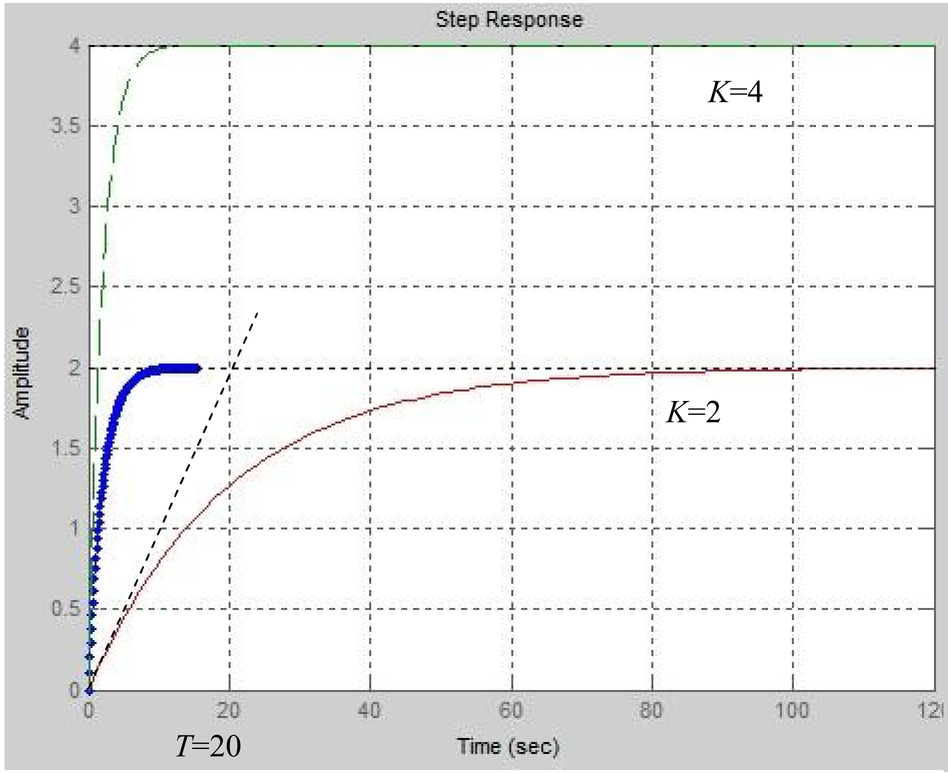
$$G_1 = \frac{2}{2s+1}, \quad G_2 = \frac{2}{20s+1}, \quad G_3 = \frac{4}{2s+1}$$

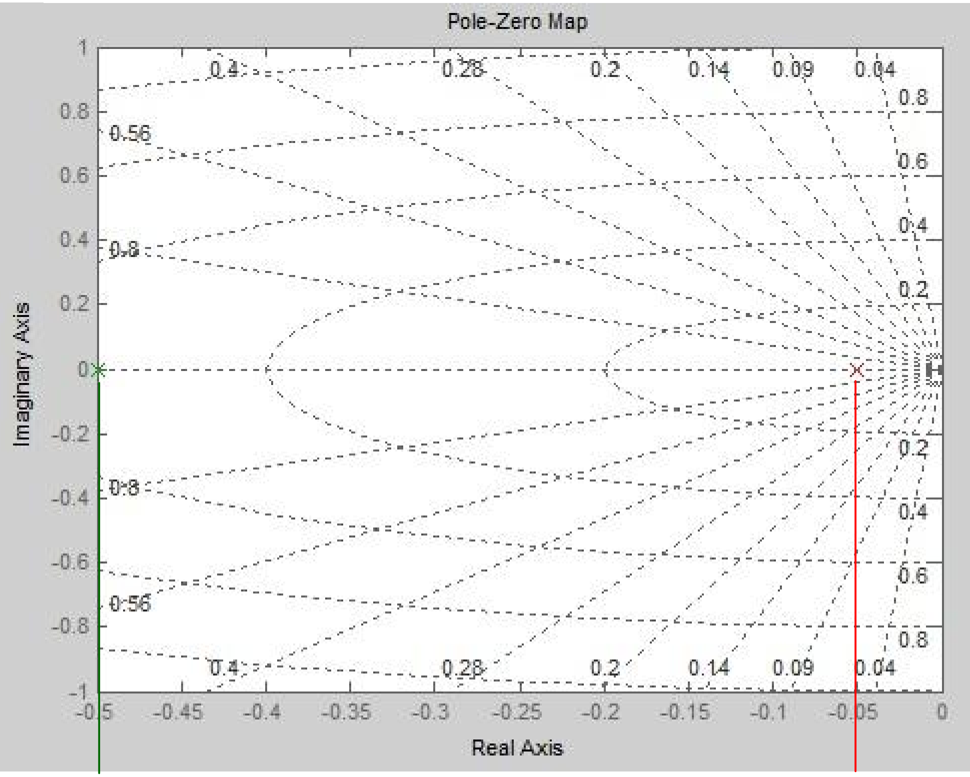
$K = 2, T = 2;$ $K = 2, T = 20;$ $K = 4, T = 2$

$s_1 = -0.5;$ $s_1 = -0.05;$ $s_1 = -0.5$



u(t)=1*(t)*





s_1=-0,5 s_1=-0,05

Przykład

$$G_1 = \frac{2}{2s+1},\quad G_2 = \frac{2}{20s+1},\quad G_3 = \frac{4}{2s+1}$$

$$G(j\omega) = \frac{K}{1+j\omega T} = \frac{K}{1+\omega^2 T^2} - j\frac{K\omega T}{1+\omega^2 T^2}$$

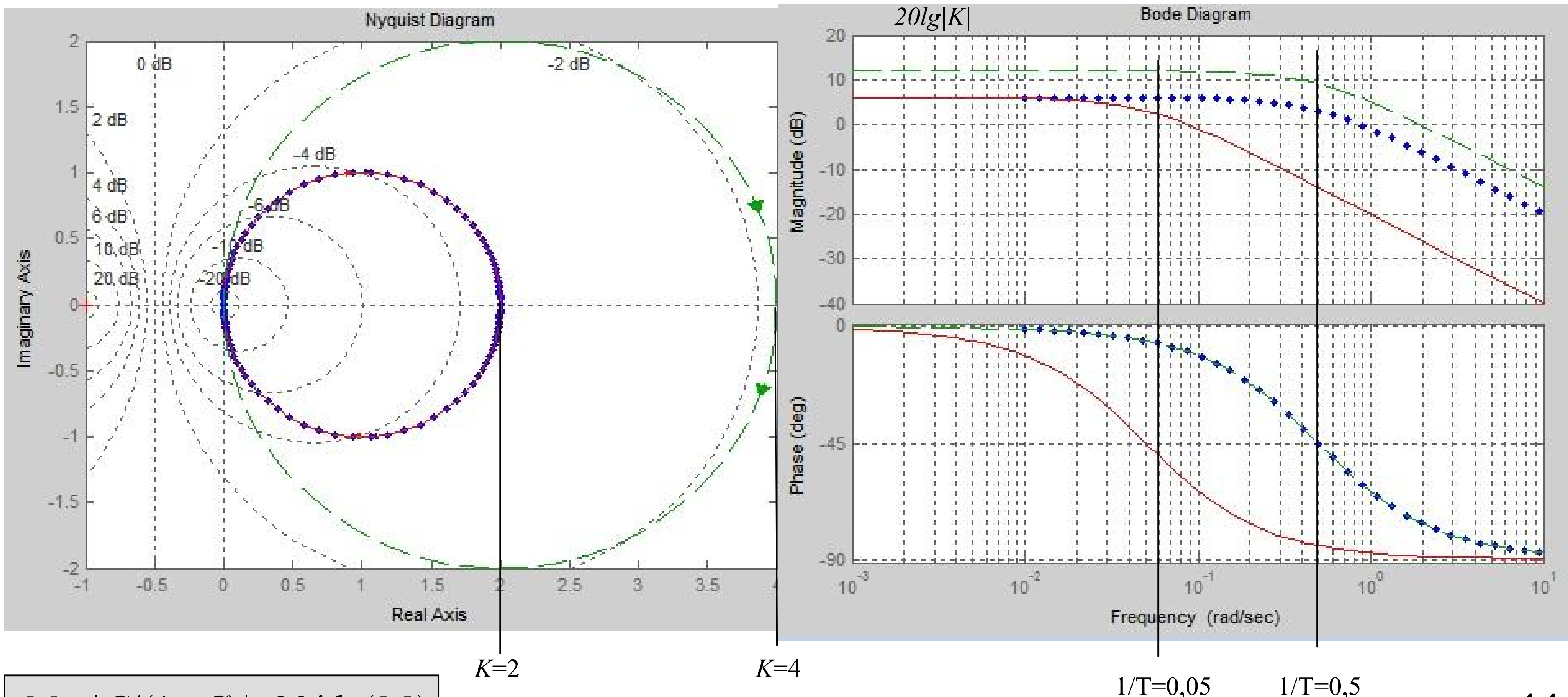
$$P(\omega) = \frac{K}{1+\omega^2 T^2}$$

$$Q(\omega) = -\frac{K\omega T}{1+\omega^2 T^2}$$

$K = 2, T = 2;\quad K = 2, T = 20;\quad K = 4, T = 2$

$20\lg 2 \approx 6{,}02\quad 20\lg 2 \approx 6{,}02\quad 20\lg 4 \approx 12{,}04$

$1/T = 0{,}5\quad 1/T = 0{,}05\quad 1/T = 0{,}5$



$M_z = |G/(1+G)|,\ 20*lg(M_z)$