B. Rozwiązania (odpowiedzi) równań I i II rzędu

B.1. Rozwiązania równania pierwszego rzędu

B.1.1 Odpowiedź skokowa i impulsowa równania pierwszego rzędu:

$$a_1\dot{x}+a_0x=bu \qquad \text{(VIII-2)}$$

Odpowiedź skokowa: $u(t)=1(t)$	Odpowiedź impulsowa: $u(t)=\delta(t)$
I. Rozwiązanie swobodne:	
$a_1\lambda+a_0=0 \;\rightarrow\; \lambda=-a_0/a_1$	
$x_s(t)=Ae^{\lambda t}$	
II. Rozwiązanie wymuszone	
Dla $u(t)=1(t)$ → gdy t>0 to u_k=1	Dla $u(t)=\delta(t)$ → gdy t>0 to u_k=0
$a_0x_k=bu_k \;\rightarrow\; x_w(t)=x_k=\dfrac{bu_k}{a_0}=\dfrac{b}{a_0}$	$a_0x_k=bu_k \;\rightarrow\; x_w(t)=x_k=\dfrac{bu_k}{a_0}=0$
III. Rozwiązanie ogólne	
$x(t)=Ae^{\lambda t}+x_k$, gdzie $x_k=b/a_0$, $\lambda=-a_0/a_1$	$x(t)=Ae^{\lambda t}+0$, gdzie $\lambda=-a_0/a_1$
IV. Warunki początkowe: $u(0_-)=0$, $x(0_-)=0$	
$0=A\cdot 1+x_k \rightarrow A=-x_k$	
$x(t)=-x_ke^{\lambda t}+x_k=x_k\left(1-e^{\lambda t}\right) \;\rightarrow\; x(0)=0$	
$\dot{x}(t)=-\lambda x_ke^{\lambda t}=-\dfrac{-a_0}{a_1}\dfrac{b}{a_0}e^{\lambda t}=\dfrac{b}{a_1}e^{\lambda t}$	$x(t)=\dfrac{b}{a_1}e^{\lambda t} \;\rightarrow\; x(0)=\dfrac{b}{a_1}$

B.1.2 Odpowiedź częstotliwościowa pierwszego rzędu:

B.2. Rozwiązania dla układu oscylacyjnego

B.2.1 Odpowiedź skokowa układu oscylacyjnego

$\ddot{x}+2\xi\omega_n\dot{x}+\omega_n^2 x=bu$ (w równaniu znormalizowanym $b=\omega_n^2$)

gdy $\xi^2 \ge 1$	gdy $\xi^2 < 1$
I. Rozwiązanie swobodne:	
$\lambda^2+2\xi\omega_n\lambda+\omega_n^2=0$	
$\lambda_{1,2}=-\xi\omega_n\pm\omega_n\sqrt{\xi^2-1}$ $\lambda_{1,2}=\alpha_{1,2}\pm j0$, gdzie $\begin{cases}\alpha_1=-\xi\omega_n+\omega_n\sqrt{\xi^2-1}\\ \alpha_2=-\xi\omega_n-\omega_n\sqrt{\xi^2-1}\end{cases}$	$\begin{cases}\lambda_1=-\xi\omega_n+\omega_n\sqrt{\xi^2-1}=-\xi\omega_n+j\omega_n\sqrt{1-\xi^2}\\ \lambda_2=-\xi\omega_n-\omega_n\sqrt{\xi^2-1}=-\xi\omega_n-j\omega_n\sqrt{1-\xi^2}\end{cases}$ $\lambda_{1,2}=\alpha\pm j\omega_r$, gdzie $\alpha=-\xi\omega_n$, $\omega_r=\omega_n\sqrt{1-\xi^2}$ $\lambda_{1,2}=-\sigma\pm j\omega_r$, gdzie $\sigma=\xi\omega_n$, $\omega_r=\omega_n\sqrt{1-\xi^2}$ *[Greblicki]*
$x_s(t)=A_1e^{\alpha_1 t}+A_2e^{\alpha_2 t}$	a) $x_s(t)=A_1e^{(\alpha+j\omega_r)t}+A_2e^{(\alpha-j\omega_r)t}$ b) $x_s(t)=e^{\alpha t}(B_1\cos\omega_r t+B_2\sin\omega_r t)$ c) $x_s(t)=Ae^{\alpha t}\sin(\omega t+\varphi_1)$, $\varphi_1=arc\,tg(B_1/B_2)$ $x_s(t)=Ae^{\alpha t}\cos(\omega_r t-\varphi_2)$, $\varphi_2=arc\,tg(B_2/B_1)$
II. Rozwiązanie wymuszone dla skoku jednostkowego $u(t)=1(t)$ → gdy $t>0$ to $u_k=1$	
$\omega_n^2 x_k=bu_k \longrightarrow x_w(t)=x_k=\frac{bu_k}{\omega_n^2}=\frac{b}{\omega_n^2}$	
III. Rozwiązanie ogólne	
$x(t)=x_k+A_1e^{\alpha_1 t}+A_2e^{\alpha_2 t}$ gdzie $x_k=b/\omega_n^2$, $\begin{cases}\alpha_1=-\xi\omega_n+\omega_n\sqrt{\xi^2-1}\\ \alpha_2=-\xi\omega_n-\omega_n\sqrt{\xi^2-1}\end{cases}$	a) $x(t)=x_k+A_1e^{(\alpha+j\omega_r)t}+A_2e^{(\alpha-j\omega_r)t}$ b) $x(t)=x_k+e^{\alpha t}(B_1\cos\omega_r t+B_2\sin\omega_r t)$ c) $x(t)=x_k+Ae^{\alpha t}\sin(\omega_r t+\varphi_1)$, $\varphi_1=\mathrm{arctg}\left(\frac{-\omega_r}{\alpha}\right)$ $x(t)=x_k+Ae^{\alpha t}\cos(\omega_r t-\varphi_2)$, $\varphi_2=\mathrm{arctg}\left(\frac{\alpha}{-\omega_r}\right)$ gdzie $x_k=b/\omega_n^2$, $\alpha=-\xi\omega_n$, $\omega_r=\omega_n\sqrt{1-\xi^2}$
IV. Warunki początkowe dla odpowiedzi skokowej: $u(0_-)=0$, $x(0_-)=0$, $\dot{x}(0_-)=0$	
Rozwiązanie 1: $x(t)$= $x_k\left(1-\frac{\alpha_2}{\alpha_2-\alpha_1}e^{\alpha_1 t}-\frac{\alpha_1}{\alpha_1-\alpha_2}e^{\alpha_2 t}\right)$ gdzie $x_k=b/\omega_n^2$, $\begin{cases}\alpha_1=-\xi\omega_n+\omega_n\sqrt{\xi^2-1}\\ \alpha_2=-\xi\omega_n-\omega_n\sqrt{\xi^2-1}\end{cases}$	Rozwiązanie 2a: $x(t)$= $x_k\left(1+\frac{\alpha-j\omega_r}{2j\omega_r}e^{(\alpha+j\omega_r)t}-\frac{\alpha+j\omega_r}{2j\omega_r}e^{(\alpha-j\omega_r)t}\right)$ Rozwiązanie 2b: $x(t)$= $x_k\left(1-e^{\alpha t}\left(\cos\omega_r t-\frac{\alpha}{\omega_r}\sin\omega_r t\right)\right)$ Rozwiązanie 2c: $x(t)$= $x_k\left(1-\frac{\sqrt{\alpha^2+\omega_r^2}}{\omega_r}e^{\alpha t}\sin(\omega_r t+\varphi_1)\right)$, $\varphi_1=\mathrm{arctg}\left(\frac{-\omega_r}{\alpha}\right)$ $x_k\left(1-\frac{\sqrt{\alpha^2+\omega_r^2}}{\omega_r}e^{\alpha t}\cos(\omega_r t-\varphi_2)\right)$, $\varphi_2=\mathrm{arctg}\left(\frac{\alpha}{-\omega_r}\right)$
Rozwiązanie 1: $x(t)$= $x_k\left(1-\frac{\xi+\sqrt{\xi^2-1}}{2\sqrt{\xi^2-1}}e^{\left(-\xi\omega_n+\omega_n\sqrt{\xi^2-1}\right)t}-\frac{-\xi+\sqrt{\xi^2-1}}{2\sqrt{\xi^2-1}}e^{\left(-\xi\omega_n-\omega_n\sqrt{\xi^2-1}\right)t}\right)$ gdzie $x_k=b/\omega_n^2$	Rozwiązanie 2a: $x(t)$= $x_k\left(1+\frac{-\xi-j\sqrt{1-\xi^2}}{2j\sqrt{1-\xi^2}}e^{(-\xi\omega_n+j\omega_n\sqrt{1-\xi^2})t}-\frac{-\xi+j\sqrt{1-\xi^2}}{2j\sqrt{1-\xi^2}}e^{(-\xi\omega_n-j\omega_n\sqrt{1-\xi^2})t}\right)$ Rozwiązanie 2b: $x(t)$= $x_k\left(1-e^{-\xi\omega_n t}\left(\cos(\omega_n\sqrt{1-\xi^2}t)+\frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_n\sqrt{1-\xi^2}t)\right)\right)$ Rozwiązanie 2c: $x(t)$= $x_k\left(1-\frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin(\omega_n\sqrt{1-\xi^2}t+\varphi_1)\right)$, $\varphi_1=\mathrm{arctg}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$ $x_k\left(1-\frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\cos\left(\omega_n\sqrt{1-\xi^2}t-\varphi_2\right)\right)$, $\varphi_2=\mathrm{arctg}\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)$

Rozwiązanie 1: $x(t)=x_k+A_1e^{\alpha_1 t}+A_2e^{\alpha_2 t}$

$$\begin{cases}x(t)=A_1e^{\alpha_1 t}+A_2e^{\alpha_2 t}+x_k\\ \dot{x}(t)=\alpha_1A_1e^{\alpha_1 t}+\alpha_2A_2e^{\alpha_2 t}\end{cases}$$

Z warunków początkowych w chwili $t=0_-$ jest:

$$\begin{cases}0=A_1+A_2+x_k\\0=\alpha_1A_1+\alpha_2A_2\end{cases}\rightarrow\begin{cases}x_k=-(A_1+A_2)\\A_2=\dfrac{-\alpha_1A_1}{\alpha_2}\end{cases}\rightarrow\begin{cases}x_k=-\left(A_1-\dfrac{\alpha_1A_1}{a_2}\right)\\A_2=\dfrac{-\alpha_1A_1}{\alpha_2}\end{cases}\rightarrow\begin{cases}\dfrac{-A_1(\alpha_2-\alpha_1)}{\alpha_2}=x_k\\A_2=\dfrac{-\alpha_1A_1}{\alpha_2}\end{cases}\rightarrow$$

$$\rightarrow\begin{cases}A_1=x_k\dfrac{-\alpha_2}{\alpha_2-\alpha_1}\\A_2=x_k\dfrac{-\alpha_1}{\alpha_1-\alpha_2}\end{cases}\rightarrow\begin{cases}A_1=x_k\dfrac{-\left(-\xi\omega_n-\omega_n\sqrt{\xi^2-1}\right)}{-\xi\omega_n-\omega_n\sqrt{\xi^2-1}-\left(-\xi\omega_n+\omega_n\sqrt{\xi^2-1}\right)}\\A_2=x_k\dfrac{-\left(-\xi\omega_n+\omega_n\sqrt{\xi^2-1}\right)}{-\xi\omega_n+\omega_n\sqrt{\xi^2-1}-\left(-\xi\omega_n-\omega_n\sqrt{\xi^2-1}\right)}\end{cases}\rightarrow\begin{cases}A_1=-x_k\dfrac{\xi+\sqrt{\xi^2-1}}{2\sqrt{\xi^2-1}}\\A_2=-x_k\dfrac{-\xi+\sqrt{\xi^2-1}}{2\sqrt{\xi^2-1}}\end{cases}$$

$$x(t)=x_k\left(1-\frac{\alpha_2}{\alpha_2-\alpha_1}e^{\alpha_1 t}-\frac{\alpha_1}{\alpha_1-\alpha_2}e^{\alpha_2 t}\right)=x_k\left(1-\frac{\xi+\sqrt{\xi^2-1}}{2\sqrt{\xi^2-1}}e^{\left(-\xi\omega_n+\omega_n\sqrt{\xi^2-1}\right)t}-\frac{-\xi+\sqrt{\xi^2-1}}{2\sqrt{\xi^2-1}}e^{\left(-\xi\omega_n-\omega_n\sqrt{\xi^2-1}\right)t}\right)$$

Rozwiązanie 2a: $x(t)=x_k+A_1e^{(\alpha+j\omega_r)t}+A_2e^{(\alpha-j\omega_r)t}$

$$\begin{cases}x(t)=A_1e^{(\alpha+j\omega_r)t}+A_2e^{(\alpha-j\omega_r)t}+x_k\\ \dot{x}(t)=(\alpha+j\omega_r)A_1e^{(\alpha+j\omega_r)t}+(\alpha-j\omega_r)A_2e^{(\alpha-j\omega_r)t}\end{cases}$$

Z warunków początkowych w chwili $t=0_-$ jest:

$$\begin{cases}0=A_1+A_2+x_k\\0=(\alpha+j\omega_r)A_1+(\alpha-j\omega_r)A_2\end{cases}\rightarrow\begin{cases}A_1=-A_2-x_k\\0=(\alpha+j\omega_r)(-A_2-x_0)+(\alpha-j\omega_r)A_2\end{cases}\rightarrow\begin{cases}A_1=-A_2-x_k\\(\alpha+j\omega_r)x_k=A_2(-\alpha-j\omega_r+\alpha-j\omega_r)\end{cases}\rightarrow$$

$$\rightarrow\begin{cases}A_1=-A_2-x_k\\(\alpha+j\omega_r)x_k=-2j\omega_rA_2\end{cases}\rightarrow\begin{cases}A_1=x_k\dfrac{\alpha+j\omega_r}{2j\omega_r}-x_k=x_k\dfrac{\alpha+j\omega_r-2j\omega_r}{2j\omega_r}\\A_2=-x_k\dfrac{\alpha+j\omega_r}{2j\omega_r}\end{cases}\rightarrow\begin{cases}A_1=x_k\dfrac{\alpha-j\omega_r}{2j\omega_r}\\A_2=-x_k\dfrac{\alpha+j\omega_r}{2j\omega_r}\end{cases}$$

$$x(t)=x_k+x_k\frac{\alpha-j\omega_r}{2j\omega_r}e^{(\alpha+j\omega_r)t}-x_k\frac{\alpha+j\omega_r}{2j\omega_r}e^{(\alpha-j\omega_r)t}\rightarrow x(t)=x_k\left(1+\frac{\alpha-j\omega_r}{2j\omega_r}e^{(\alpha+j\omega_r)t}-\frac{\alpha+j\omega_r}{2j\omega_r}e^{(\alpha-j\omega_r)t}\right)$$

$$x(t)=x_k+x_k\frac{-\xi\omega_n-j\omega_n\sqrt{1-\xi^2}}{2j\omega_n\sqrt{1-\xi^2}}e^{(-\xi\omega_n+j\omega_n\sqrt{1-\xi^2})t}-x_k\frac{-\xi\omega_n+j\omega_n\sqrt{1-\xi^2}}{2j\omega_n\sqrt{1-\xi^2}}e^{(-\xi\omega_n-j\omega_n\sqrt{1-\xi^2})t}$$

$$x(t)=x_k\left(1+\frac{-\xi-j\sqrt{1-\xi^2}}{2j\sqrt{1-\xi^2}}e^{(-\xi\omega_n+j\omega_n\sqrt{1-\xi^2})t}-\frac{-\xi+j\sqrt{1-\xi^2}}{2j\sqrt{1-\xi^2}}e^{(-\xi\omega_n-j\omega_n\sqrt{1-\xi^2})t}\right)$$

Rozwiązanie 2b: $x(t)=x_k+e^{\alpha t}\left(B_1\cos\omega_rt+B_2\sin\omega_rt\right)$

$$\begin{cases}x(t)=e^{\alpha t}\left(B_1\cos\omega_rt+B_2\sin\omega_rt\right)+x_k\\ \dot{x}(t)=\alpha e^{\alpha t}\left(B_1\cos\omega_rt+B_2\sin\omega_rt\right)+e^{\alpha t}\omega_r\left(-B_1\sin\omega_rt+B_2\cos\omega_rt\right)\end{cases}$$

Z warunków początkowych w chwili $t=0_-$ jest:

$$\begin{cases}0=B_1+x_k\\0=\alpha B_1+B_2\omega_r\end{cases}\rightarrow\begin{cases}B_1=-x_k\\B_2=\dfrac{-\alpha B_1}{\omega_r}=\dfrac{\alpha x_k}{\omega_r}\end{cases}\rightarrow\begin{cases}B_1=-x_k\\B_2=\dfrac{-\xi\omega_nx_k}{\omega_n\sqrt{1-\xi^2}}=\dfrac{-\xi x_k}{\sqrt{1-\xi^2}}\end{cases}$$

$$x(t)=x_k+e^{\alpha t}\left(-x_k\cos\omega_rt+\frac{\alpha x_k}{\omega_r}\sin\omega_rt\right)\rightarrow x(t)=x_k\left[1-e^{\alpha t}\left(\cos\omega_rt-\frac{\alpha}{\omega_r}\sin\omega_rt\right)\right]$$

$$x(t)=x_k+e^{-\xi\omega_nt}\left(-x_k\cos\omega_rt-\frac{\xi x_k}{\sqrt{1-\xi^2}}\sin\omega_rt\right)\rightarrow x(t)=x_k\left[1-e^{-\xi\omega_nt}\left(\cos(\omega_n\sqrt{1-\xi^2}t)+\frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_n\sqrt{1-\xi^2}t)\right)\right]$$

Rozwiązanie 2c₁: $x(t) = x_k + Ae^{\alpha t}\sin(\omega_r t + \varphi_1)$,

$$\begin{cases} x(t) = Ae^{\alpha t}\sin(\omega_r t + \varphi_1) + x_k \\ \dot{x}(t) = \alpha Ae^{\alpha t}\sin(\omega_r t + \varphi_1) + Ae^{\alpha t}\omega_r\cos(\omega_r t + \varphi_1) \end{cases}$$

Z warunków początkowych w chwili $t=0_-$ jest:

$$\begin{cases} 0 = A\sin(\varphi_1) + x_k \\ 0 = A\alpha\sin(\varphi_1) + A\omega_r\cos(\varphi_1) \end{cases} \to \begin{cases} 0 = A\sin\varphi_1 + x_k \\ 0 = \alpha\sin\varphi_1 + \omega_r\cos\varphi_1 \end{cases} \to \begin{cases} \sin\varphi_1 = -x_k / A \\ 0 = \alpha\sin\varphi_1 + \omega_r\cos\varphi_1 \end{cases}$$

1° $0 = \alpha\sin\varphi_1 + \omega_r\cos\varphi_1 \to \alpha\sin\varphi_1 = -\omega_r\cos\varphi_1 \to \operatorname{tg}\varphi_1 = \frac{-\omega_r}{\alpha}$ lub $\operatorname{ctg}\varphi_1 = \frac{\alpha}{-\omega_r}$

$$\varphi_1 = \operatorname{arctg}\left(\frac{-\omega_r}{\alpha}\right) \to \varphi_1 = \operatorname{arctg}\left(\frac{-\omega_n\sqrt{1-\xi^2}}{-\xi\omega_n}\right) \to \varphi_1 = \operatorname{arctg}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right),$$

2° $\alpha\sin\varphi_1 = -\omega_r\cos\varphi_1 \to \alpha^2\sin^2\varphi_1 = (-\omega_r)^2\cos^2\varphi_1 \to \alpha^2\sin^2\varphi_1 = (-\omega_r)^2(1-\sin^2\varphi_1) \to (\alpha^2+\omega_r^2)\sin^2\varphi_1 = (-\omega_r)^2$

$$\to (\alpha^2+\omega_r^2)\frac{x_k^2}{A^2} = (-\omega_r)^2 \to A = x_k\frac{\sqrt{\alpha^2+\omega_r^2}}{-\omega_r} \to A = -x_k\frac{\sqrt{(-\xi\omega_n)^2+\omega_n^2(1-\xi^2)}}{\omega_n\sqrt{1-\xi^2}} \to A = -x_k\frac{1}{\sqrt{1-\xi^2}}$$

$$x(t) = x_k - x_k\frac{\sqrt{\alpha^2+\omega_r^2}}{\omega_r}e^{\alpha t}\sin(\omega_r t + \varphi_1) \to \boxed{x(t) = x_k\left[1 - \frac{\sqrt{\alpha^2+\omega_r^2}}{\omega_r}e^{\alpha t}\sin(\omega_r t + \varphi_1)\right]},\ \varphi_1 = \operatorname{arctg}\left(\frac{-\omega_r}{\alpha}\right)$$

$$x(t) = x_k - x_k\frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin(\omega_n\sqrt{1-\xi^2}\,t + \varphi_1) \to \boxed{x(t) = x_k\left[1 - \frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin(\omega_n\sqrt{1-\xi^2}\,t + \varphi_1)\right]},\ \varphi_1 = \operatorname{arctg}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

Rozwiązanie 2c₂: $x(t) = x_k + Ae^{\alpha t}\cos(\omega_r t - \varphi_2)$

$$\begin{cases} x(t) = Ae^{\alpha t}\cos(\omega_r t - \varphi_2) + x_k \\ \dot{x}(t) = \alpha Ae^{\alpha t}\cos(\omega_r t - \varphi_2) - Ae^{\alpha t}\omega_r\sin(\omega_r t - \varphi_2) \end{cases}$$

Z warunków początkowych w chwili $t=0_-$ jest:

$$\begin{cases} 0 = A\cos(-\varphi_2) + x_k \\ 0 = A\alpha\cos(-\varphi_2) - A\omega_r\sin(-\varphi_2) \end{cases} \to \begin{cases} 0 = A\cos\varphi_2 + x_k \\ 0 = \alpha\cos\varphi_2 + \omega_r\sin\varphi_2 \end{cases} \to \begin{cases} \cos\varphi_2 = -x_k / A \\ 0 = \alpha\cos\varphi_2 + \omega_r\sin\varphi_2 \end{cases}$$

1° $0 = \alpha\cos\varphi_2 + \omega_r\sin\varphi_2 \to \alpha\cos\varphi_2 = -\omega_r\sin\varphi_2 \to tg\varphi_2 = \frac{\alpha}{-\omega_r} \to$

$$\varphi_2 = \operatorname{arctg}\left(\frac{\alpha}{-\omega_r}\right) \to \varphi_2 = \operatorname{arctg}\left(\frac{-\xi\omega_n}{-\omega_n\sqrt{1-\xi^2}}\right) \to \varphi_2 = \operatorname{arctg}\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)$$

2° $-\alpha\cos\varphi_2 = \omega_r\sin\varphi_2 \to \alpha^2\cos^2\varphi_2 = (-\omega_r)^2\sin^2\varphi_2 \to \alpha^2\cos^2\varphi_2 = (-\omega_r)^2(1-\cos^2\varphi_2) \to (\alpha^2+\omega_r^2)\cos^2\varphi_2 = (-\omega_r)^2$

$$\to (\alpha^2+\omega_r^2)\frac{x_k^2}{A^2} = (-\omega_r)^2 \to A = x_k\frac{\sqrt{\alpha^2+\omega_r^2}}{-\omega_r} \to A = x_k\frac{\sqrt{(-\xi\omega_n)^2+\omega_n^2(1-\xi^2)}}{-\omega_n\sqrt{1-\xi^2}} \to A = -x_k\frac{1}{\sqrt{1-\xi^2}}$$

$$x(t) = x_k - x_k\frac{\sqrt{\alpha^2+\omega_r^2}}{\omega_r}e^{\alpha t}\cos(\omega_r t - \varphi_2) \to \boxed{x(t) = x_k\left[1 - \frac{\sqrt{\alpha^2+\omega_r^2}}{\omega_r}e^{\alpha t}\cos(\omega_r t - \varphi_2)\right]},\ \text{gdzie}\ \varphi_2 = \operatorname{arctg}\left(\frac{\alpha}{-\omega_r}\right),$$

$$x(t) = x_k - x_k\frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\cos(\omega_n\sqrt{1-\xi^2}\,t - \varphi_2) \to \boxed{x(t) = x_k\left[1 - \frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\cos(\omega_n\sqrt{1-\xi^2}\,t - \varphi_2)\right]},\ \text{gdzie}\ \varphi_2 = \operatorname{arctg}\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)$$

Sprawdzenie prostych wzorów dla $\xi^2 < 1$, x_k=1 [skrypt: testy_RozwiązanieRR]:

	wartości wyliczone z warunków początkowych (rozwiązania powyżej: 1,2a,2b,2c)	
a) $1+A_1 e^{(\alpha+j\omega)t}+A_2 e^{(\alpha-j\omega)t}$	$A_1=\frac{\alpha-j\omega_r}{2j\omega_r}$, $A_2=-\frac{\alpha+j\omega_r}{2j\omega_r}$	$1+\frac{\alpha-j\omega_r}{2j\omega_r}e^{(\alpha+j\omega_r)t}-\frac{\alpha+j\omega_r}{2j\omega_r}e^{(\alpha-j\omega_r)t}$
b) $1+e^{\alpha t}(B_1\cos\omega_r t+B_2\sin\omega_r t)$ $B_1=A_1+A_2$, $B_2=j(A_1-A_2)$	$B_1=-1$, $B_2=\frac{\alpha}{\omega_r}$	$1+e^{\alpha t}\left(-\cos\omega_r t+\frac{\alpha}{\omega_r}\sin\omega_r t\right)$
c₁) $1+Ae^{\alpha t}\sin(\omega_r t+\varphi_1)$ $A=\sqrt{B_1^2+B_2^2}$, $\varphi_1=arc\,tg(B_1/B_2)$	$A=-\frac{\sqrt{\alpha^2+\omega_r^2}}{\omega_r}$, $\varphi_1=\mathrm{arc\,tg}\left(\frac{-\omega_r}{\alpha}\right)$	$1-\frac{\sqrt{\alpha^2+\omega_r^2}}{\omega_r}e^{\alpha t}\sin(\omega_r t+\varphi_1)$
c₂) $1+Ae^{\alpha t}\cos(\omega_r t-\varphi_2)$ $A=\sqrt{B_1^2+B_2^2}$, $\varphi_2=arc\,tg(B_2/B_1)$,	$A=-\frac{\sqrt{\alpha^2+\omega_r^2}}{\omega_r}$, $\varphi_2=\mathrm{arc\,tg}\left(\frac{-\alpha}{\omega_r}\right)$	$1-\frac{\sqrt{\alpha^2+\omega_r^2}}{\omega_r}e^{\alpha t}\cos(\omega_r t-\varphi_2)$

Sprawdzenie zależności:

a → b) $$B_1=A_1+A_2=\frac{\alpha-j\omega_r}{2j\omega_r}-\frac{\alpha+j\omega_r}{2j\omega_r}=\frac{-2j\omega_r}{2j\omega_r}=-1,$$

$$B_2=j(A_1-A_2)=j\frac{\alpha-j\omega_r}{2j\omega_r}+j\frac{\alpha+j\omega_r}{2j\omega_r}=\frac{2\alpha}{2\omega_r}=\frac{\alpha}{\omega_r}$$

b → c) $$A=\sqrt{B_1^2+B_2^2}=\sqrt{(-1)^2+\left(\frac{\alpha}{\omega_r}\right)^2}=\sqrt{1+\frac{\alpha^2}{\omega_r^2}}=\sqrt{\frac{\alpha^2+\omega_r^2}{\omega_r^2}}$$

$$\varphi_1=arctg\left(\frac{B_1}{B_2}\right)=\mathrm{arc\,tg}\left(\frac{-1}{\alpha/\omega_r}\right)=\mathrm{arc\,tg}\left(\frac{-\omega_r}{\alpha}\right),\qquad \varphi_2=arctg\left(\frac{B_2}{B_1}\right)=\mathrm{arc\,tg}\left(\frac{\alpha/\omega_r}{-1}\right)=\mathrm{arc\,tg}\left(\frac{-\alpha}{\omega_r}\right)$$

Sprawdzenie wzorów rozwiniętych dla $\xi^2 < 1$, x_k=1 [skrypt: testy_RozwiązanieRR]:

	wartości wyliczone z warunków początkowych (rozwiązania powyżej: 1,2a,2b,2c)	
a) $1+A_1 e^{(\alpha+j\omega)t}+A_2 e^{(\alpha-j\omega)t}$	$A_1=\frac{-\xi-j\sqrt{1-\xi^2}}{2j\sqrt{1-\xi^2}}$, $A_2=-\frac{-\xi+j\sqrt{1-\xi^2}}{2j\sqrt{1-\xi^2}}$	$1+\frac{-\xi-j\sqrt{1-\xi^2}}{2j\sqrt{1-\xi^2}}e^{(\alpha+j\omega_r)t}-\frac{-\xi+j\sqrt{1-\xi^2}}{2j\sqrt{1-\xi^2}}e^{(\alpha-j\omega_r)t}$
b) $1+e^{\alpha t}(B_1\cos\omega_r t+B_2\sin\omega_r t)$ $B_1=A_1+A_2$, $B_2=j(A_1-A_2)$	$B_1=-1$; $B_2=\frac{-\xi}{\sqrt{1-\xi^2}}\rightarrow B_2=\frac{-\xi}{\sqrt{1-\xi^2}}$	$1+e^{-\xi\omega_n t}\left(-\cos(\omega_n\sqrt{1-\xi^2}t)-\frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_n\sqrt{1-\xi^2}t)\right)$
c) $1+Ae^{\alpha t}\sin(\omega_r t+\varphi_1)$ $A=\sqrt{B_1^2+B_2^2}$, $\varphi_1=arc\,tg(B_1/B_2)$	$A=\frac{-1}{\sqrt{1-\xi^2}}$, $\varphi_1=\mathrm{arc\,tg}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$	$1-\frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin(\omega_n\sqrt{1-\xi^2}t+\varphi_1)$
c) $1+Ae^{\alpha t}\cos(\omega_r t-\varphi_2)$ $A=\sqrt{B_1^2+B_2^2}$, $\varphi_2=arc\,tg(B_2/B_1)$,	$A=\frac{-1}{\sqrt{1-\xi^2}}$, $\varphi_2=\mathrm{arc\,tg}\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)$	$1-\frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\cos\left(\omega_n\sqrt{1-\xi^2}t-\varphi_2\right)$

Sprawdzenie zależności:

a → b) $$B_1=A_1+A_2=\frac{-\xi-j\sqrt{1-\xi^2}}{2j\sqrt{1-\xi^2}}-\frac{-\xi+j\sqrt{1-\xi^2}}{2j\sqrt{1-\xi^2}}=\frac{-2j\sqrt{1-\xi^2}}{2j\sqrt{1-\xi^2}}=-1,$$

$$B_2=j(A_1-A_2)=j\frac{-\xi-j\sqrt{1-\xi^2}}{2j\sqrt{1-\xi^2}}+j\frac{-\xi+j\sqrt{1-\xi^2}}{2j\sqrt{1-\xi^2}}=\frac{-2\xi}{2\sqrt{1-\xi^2}}=\frac{-\xi}{\sqrt{1-\xi^2}}$$

b → c) $$A=\sqrt{B_1^2+B_2^2}=\sqrt{(-1)^2+\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)^2}=\sqrt{1+\frac{\xi^2}{1-\xi^2}}=\sqrt{1+\frac{\xi^2}{1-\xi^2}}=\sqrt{\frac{1-\xi^2+\xi^2}{1-\xi^2}}=\frac{1}{\sqrt{1-\xi^2}}$$

$$\varphi_1=arctg\left(\frac{B_1}{B_2}\right)=\mathrm{arc\,tg}\left(-1/\frac{-\xi}{\sqrt{1-\xi^2}}\right)=\mathrm{arc\,tg}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right),\qquad \varphi_2=arctg\left(\frac{B_2}{B_1}\right)=\mathrm{arc\,tg}\left(\frac{-\xi}{\sqrt{1-\xi^2}}/(-1)\right)=\mathrm{arc\,tg}\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)$$

Dodać odp. skokowa członu znormalizowanego: $x(t)=1-\frac{\sqrt{\sigma^2+\omega^2}}{\omega_r}e^{-\sigma t}\cos(\omega t+\varphi)$, $\varphi=\mathrm{arc\,tg}\left(\frac{\omega}{\sqrt{\sigma^2+\omega^2}}\right)$

B.2.2 Odpowiedź częstotliwościowa układu oscylacyjnego