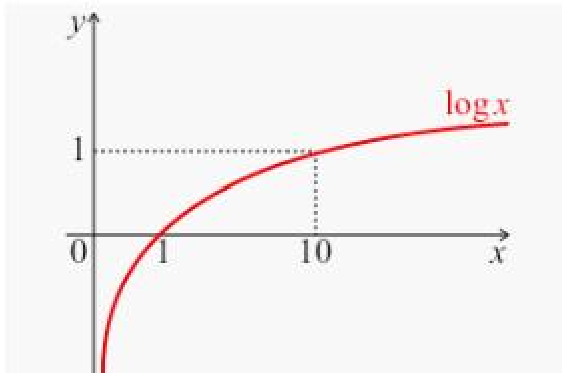
# Logarytmy

Definicja:

$$\log_a b = c \Leftrightarrow a^c = b,$$

$$a, b > 0, a \neq 1$$




Wybrane własności:

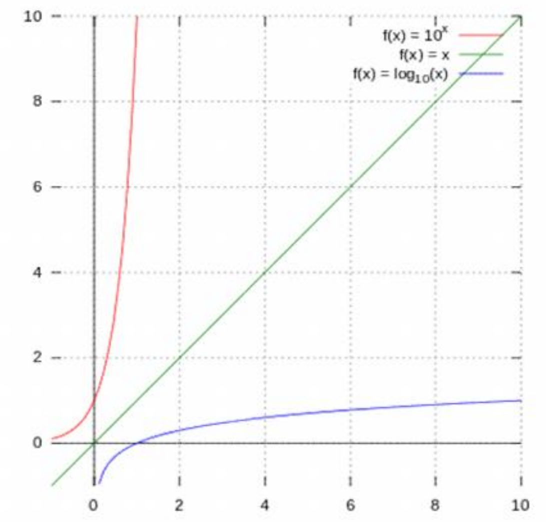
$$\log_a(bc) = \log_a b + \log_a c$$

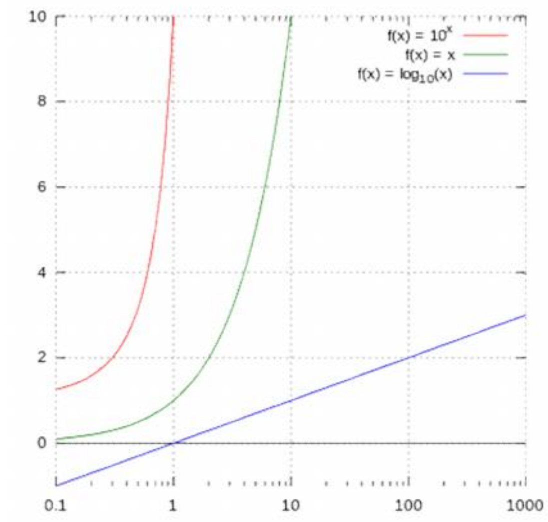
$$\log_a \frac{1}{b} = -\log_a b$$

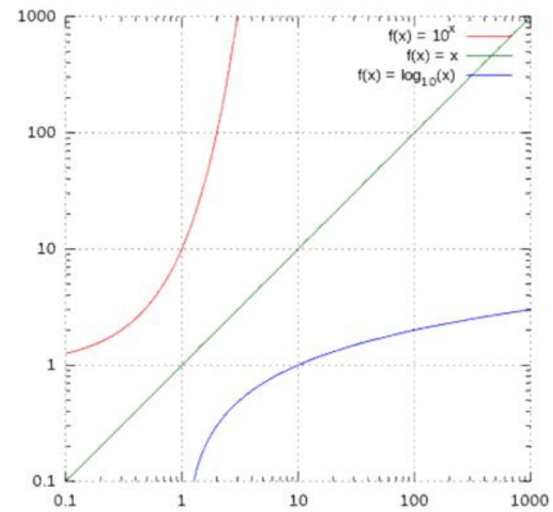
$$\log_a \frac{b}{c} = \log_a b - \log_a c$$

$$\log_a b^n = n \log_a b$$

Zastosowanie na skali logarytmicznej




$$P_B = \log_{10}\left(\frac{P}{P_0}\right)$$

**bel** (zwiększenie mocy *P* względem mocy $P_0$)

$$1\,\mathrm{dB} = \frac{1}{10}\,\mathrm{B}$$

**decybel**

$$P_{\mathrm{dB}} = 10\log_{10}\left(\frac{P}{P_0}\right)$$

$$L\,[\mathrm{dB}] = 10\log_{10}\left(\frac{A^2}{A_0^2}\right) = 20\log_{10}\left(\frac{A}{A_0}\right)$$

Amplituda sygnału jest proporcjonalna do kwadratu mocy

Przykład: Moc sygnały wyjściowego jest $10^6$ większa od sygnału wejściowego:

$$\log_{10}\left(\frac{1.000.000 \cdot P_{prg}}{P_{prg}}\right) = \log\left(10^6 \cdot \frac{P_{prg}}{P_{prg}}\right) = \log_{10}\left(10^6\right) = 6\,\mathrm{B} = 60\,\mathrm{dB}$$

## Transmitancja widmowa

G(s) = G(jω),

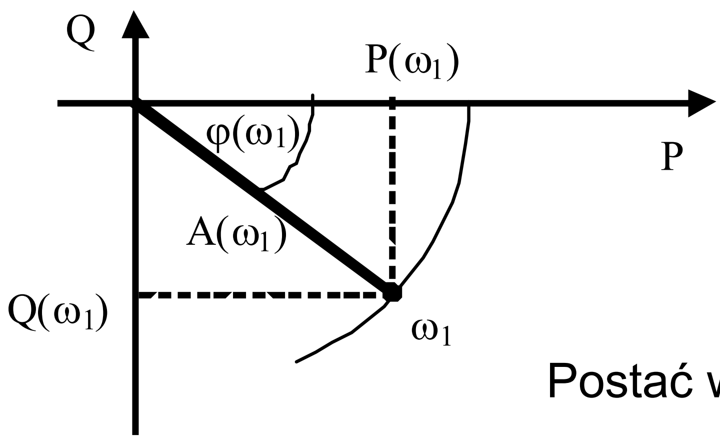
$$G(j\omega)=\frac{L(j\omega)}{M(j\omega)}=\frac{P_1(\omega)+jQ_1(\omega)}{P_2(\omega)+jQ_2(\omega)}=\frac{[P_1(\omega)+jQ_1(\omega)][P_2(\omega)-jQ_2(\omega)]}{P_2^2(\omega)+Q_2^2(\omega)}$$

$$G(j\omega)=P(\omega)+jQ(\omega)$$

$$G(j\omega)=A(\omega)e^{j\varphi(\omega)}$$

$$P(\omega)=\frac{P_1P_2+Q_1Q_2}{P_2^2+Q_2^2}\qquad Q(\omega)=\frac{P_2Q_1-P_1Q_2}{P_2^2+Q_2^2}$$

$$A(\omega)=\sqrt{\frac{P_1^2+Q_1^2}{P_2^2+Q_2^2}}=\frac{A_1}{A_2}\qquad \varphi(\omega)=ar\,\mathrm{ctg}\frac{P_2Q_1-P_1Q_2}{P_1P_2+Q_1Q_2}$$




Postać algebraiczna

$$G(j\omega)=P(\omega)+jQ(\omega)$$

$$P(\omega)=A(\omega)\cos[\varphi(\omega)]$$

$$Q(\omega)=A(\omega)\sin[\varphi(\omega)]$$

Postać wykładnicza

$$G(j\omega)=A(\omega)e^{j\varphi(\omega)}$$

$$A(\omega)=|G(j\omega)|=\sqrt{P^2(\omega)+Q^2(\omega)}$$

$$\varphi(\omega)=\arg[G(j\omega)]=ar\,\mathrm{ctg}\frac{Q(\omega)}{P(\omega)}$$

## Charakterystyki częstotliwościowe

$$G(j\omega) = \frac{L(j\omega)}{M(j\omega)} = P(\omega) + jQ(\omega) = A(\omega)e^{j\varphi(\omega)}$$

| | |
|---|---|
| ch-ka rzeczywista | - $P(\omega) = \mathrm{Re}(G(j\omega))$ |
| ch-ka urojona | - $Q(\omega) = \mathrm{Im}(G(j\omega))$ |
| ch. aplitudowo-fazowa | - Q(P) (ch.Nyquista - dla ukł. otwartych) |
| ch. amplitudowa | - $A(\omega) = \lvert G(\omega)\rvert$ |
| ch. fazowa | - $\varphi(\omega)$ |
| logarytmiczna ch. modułu | - $M(\omega)= 20 \lg A(\omega)$ |
| logarytmiczna ch. fazy | - $\varphi(\omega) = \mathrm{arctg}(Q/P)$ |
| log.ch.amplitudowo-fazowa | - $M(\varphi)$ |

### Charakterystyki członów połączonych szeregowo

$$G(j\omega) = \prod_{i=1}^{n} G_i(j\omega) = \prod \left[A_i(\omega)e^{j\varphi_i(\omega)}\right]$$

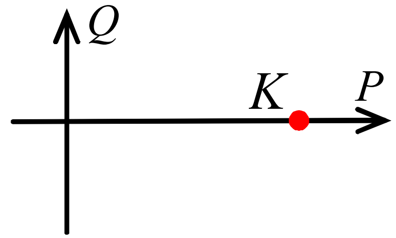
- ch. amplitudowa $$A(\omega) = \prod A_i(\omega)$$
- ch. fazowa $$\varphi(\omega) = \sum \varphi_i(\omega)$$
- logarytmiczna ch.aplitudowa $$M(\omega) = 20\lg(\prod A_i(\omega)) = \sum M_i(\omega)$$
- logarytmiczna ch. fazy $$\varphi(\omega) = \sum \varphi_i(\omega)$$

**cz. proporcjonalny:**

$$P(\omega) + jQ(\omega) = A(\omega)e^{j\varphi(\omega)}$$
$$M(\omega) = 20\lg A(\omega)$$
$$\varphi(\omega) = arctg(Q/P)$$

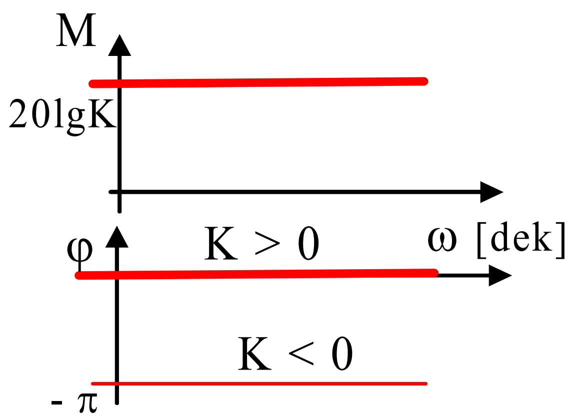
$$G(s) = K \qquad G(j\omega) = K$$

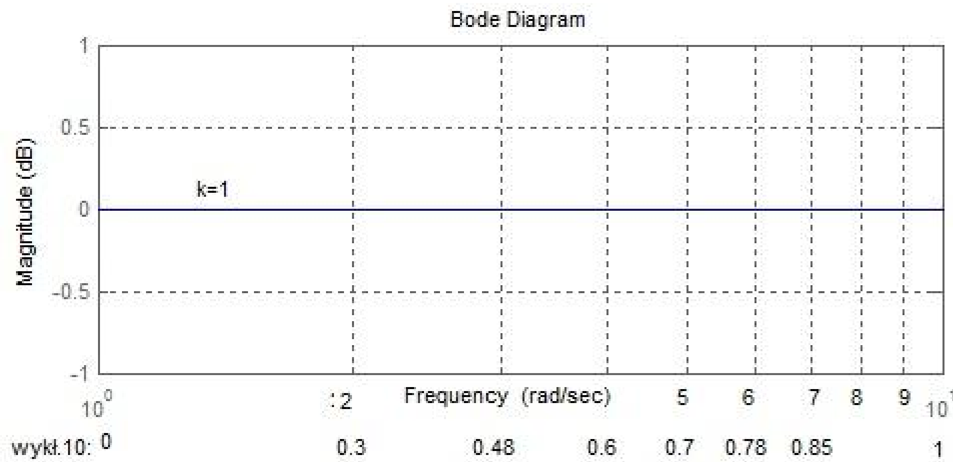
$$P(\omega) = K$$
$$Q(\omega) = 0$$




$$A(\omega) = |K|$$
$$\varphi(\omega) = 0$$

$$M(\omega) = 20\lg|K|$$

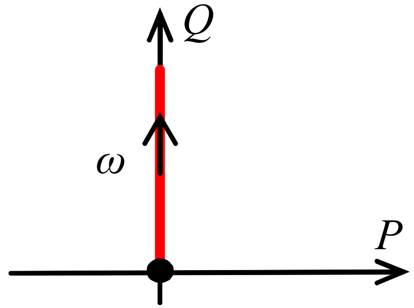


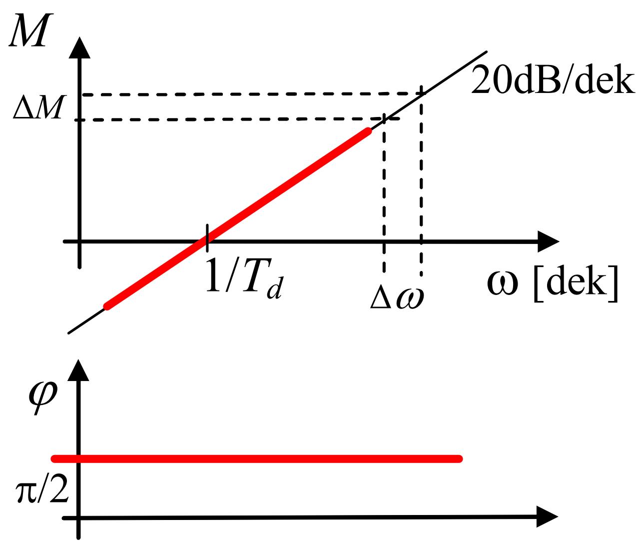
**cz. różniczkowy:**

$$G(s) = sT_d \qquad G(j\omega) = j\omega T_d$$

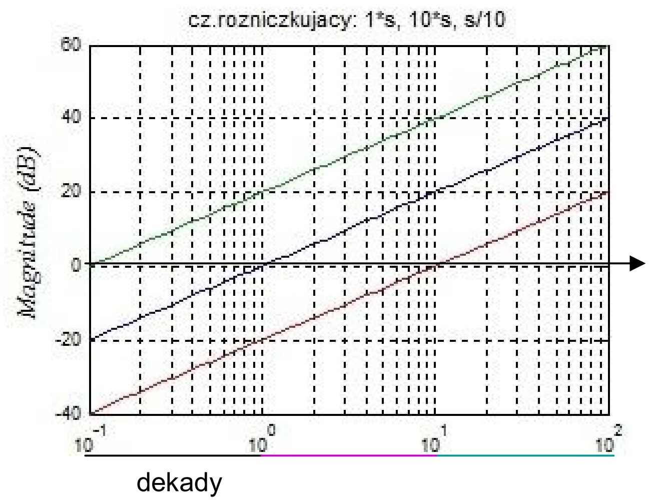
$$P(\omega) + jQ(\omega) = A(\omega)e^{j\varphi(\omega)}$$
$$M(\omega) = 20\lg A(\omega)$$
$$\varphi(\omega) = arctg(Q/P)$$




$$P(\omega) = 0$$
$$Q(\omega) = \omega T_d$$




$$A(\omega) = |j\omega T_d| = \omega T_d$$
$$\varphi(\omega) = \pi/2$$
$$M(\omega) = 20\lg|\omega T_d|$$




Legenda ???

$$M = 0 \rightarrow \omega T_d = 1 \rightarrow \omega = \frac{1}{T_d}$$

$$\frac{\Delta M}{\Delta\omega} = 20\lg(10\omega_1) - 20\lg(\omega_1) = 20\lg\frac{10\omega_1}{\omega_1} = 20\frac{dB}{dek}$$

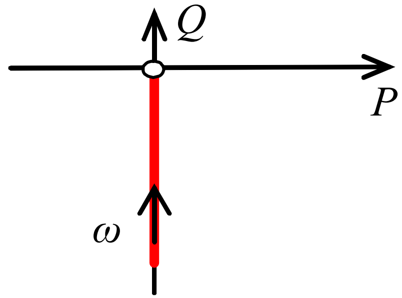
**cz. całkujący:**

$$P(\omega) + jQ(\omega) = A(\omega)e^{j\varphi(\omega)}$$
$$M(\omega) = 20\lg A(\omega)$$
$$\varphi(\omega) = arctg\left(Q/P\right)$$

$$G(s) = \frac{K}{sT_i} \qquad G(j\omega) = \frac{K}{j\omega T_i} = -j\frac{K}{\omega T_i}$$

$$P(\omega) = 0$$

$$Q(\omega) = -\frac{K}{\omega T_d}$$

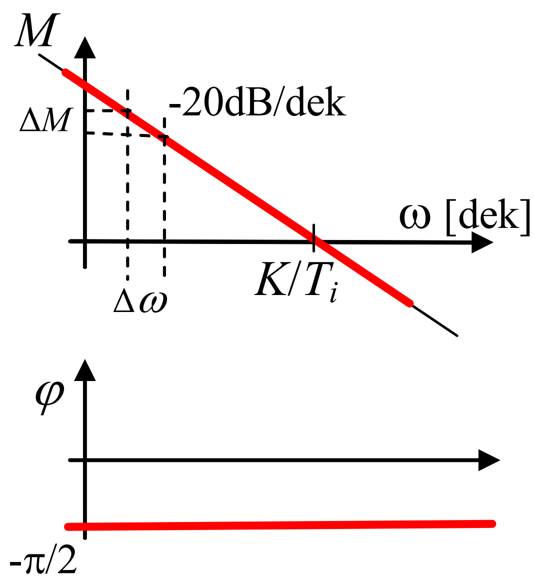


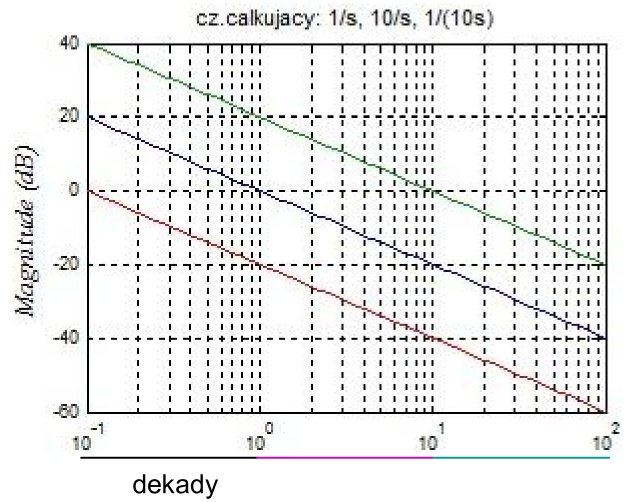

$$A(\omega) = \left|\frac{K}{j\omega T_i}\right| = \frac{K}{\omega T_i}$$

$$\varphi(\omega) = -\pi/2$$

$$M(\omega) = 20\lg\left|\frac{K}{\omega T_i}\right| =$$

$$= 20\lg K - 20\lg\omega - 20\lg T_i$$




Legenda ???

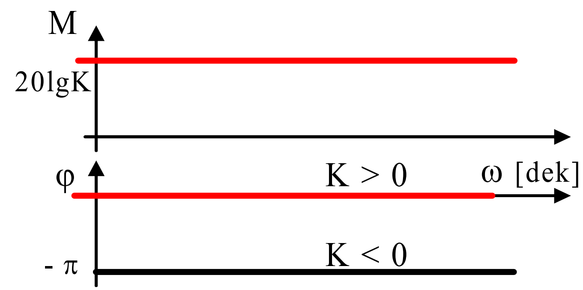
$$M = 0 \rightarrow \frac{K}{\omega T_i} = 1 \rightarrow \omega = \frac{K}{T_i}$$

$$\frac{\Delta M}{\Delta\omega} = -20\lg(10\omega_1) + 20\lg(\omega_1) = 20\lg\frac{\omega_1}{10\omega_1} = -20\frac{dB}{dek}$$

**cz. proporcjonalny:**

$$G(s) = K$$

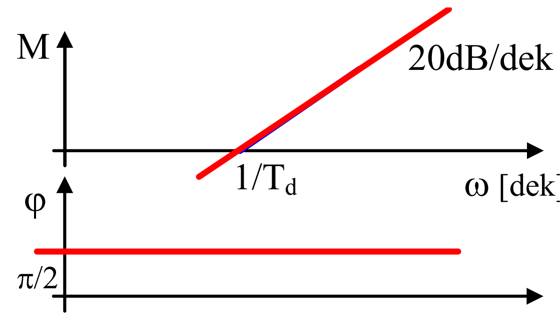
$$G(j\omega) = K$$
$$M(\omega) = 20\lg|K|$$




**cz. różniczkowy:**

$$G(s) = sT_d$$

$$G(j\omega) = j\omega T_d$$
$$M(\omega) = 20\lg|\omega T_d|$$

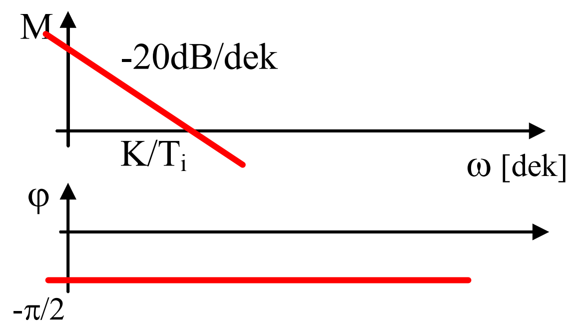



**cz. całkujący:**

$$G(s) = \frac{K}{sT_i}$$

$$G(j\omega) = \frac{K}{j\omega T_i} = -j\frac{K}{\omega T_i}$$

$$M(\omega) = 20\lg\left|\frac{K}{\omega T_i}\right|$$

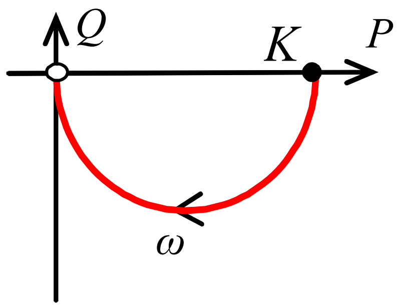


**cz. inercyjny:**

$$P(\omega) + jQ(\omega) = A(\omega)e^{j\varphi(\omega)}$$
$$M(\omega) = 20\lg A(\omega)$$
$$\varphi(\omega) = arctg(Q/P)$$

$$G(s) = \frac{K}{1+sT} \qquad G(j\omega) = \frac{K}{1+j\omega T} = \frac{K}{1+\omega^2T^2} - j\frac{K\omega T}{1+\omega^2T^2}$$

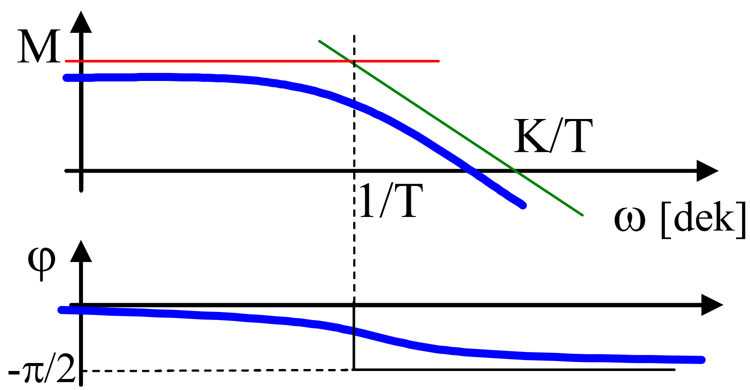
$$P(\omega) = \frac{K}{1+\omega^2T^2}$$
$$Q(\omega) = -\frac{K\omega T}{1+\omega^2T^2}$$




$$A(\omega) = \left|\frac{K}{1+j\omega T}\right|$$
$$\varphi(\omega) = arctg(-\omega T)$$

dla ω<<1/T $\quad G(j\omega) \approx K$

dla ω>>1/T $\quad G(j\omega) \approx \dfrac{K}{j\omega T}$

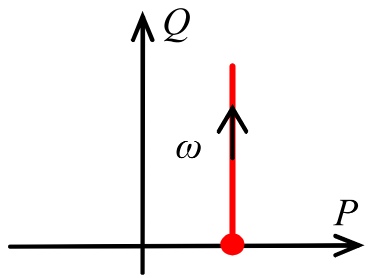


**cz. forsujący:**

$$P(\omega) + jQ(\omega) = A(\omega)e^{j\varphi(\omega)}$$
$$M(\omega) = 20\lg A(\omega)$$
$$\varphi(\omega) = arctg(Q/P)$$

$$G(s) = 1 + sT \qquad G(j\omega) = 1 + j\omega T$$

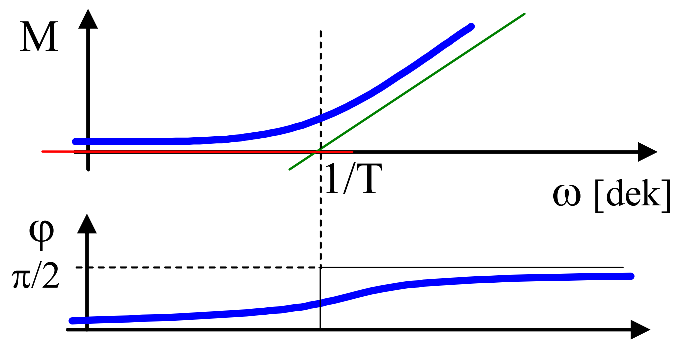
$$P(\omega) = 1$$
$$Q(\omega) = \omega T$$




$$A(\omega) = |1 + j\omega T|$$
$$\varphi(\omega) = arctg(\omega T)$$

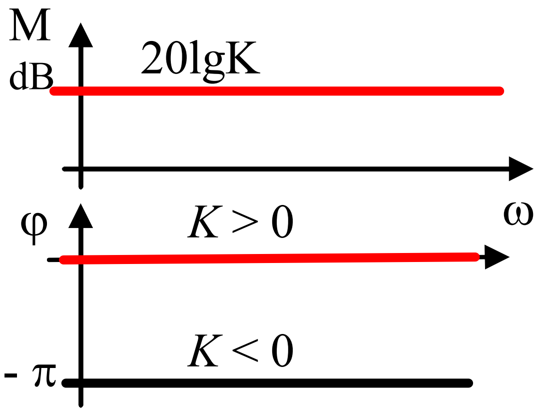
dla ω<<1/T $\quad G(j\omega) \approx 1$

dla ω>>1/T $\quad G(j\omega) \approx j\omega T$

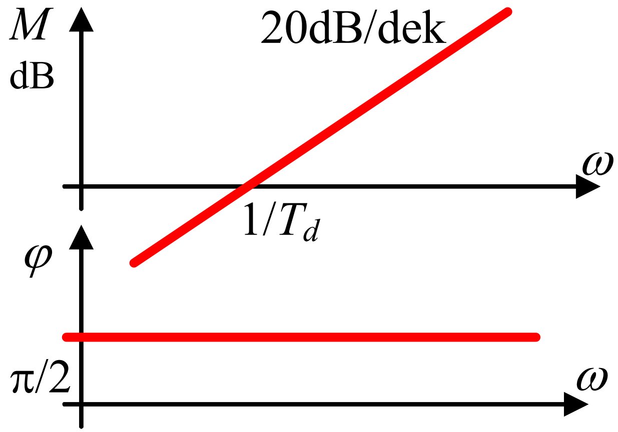


# Logarytmiczne charakterystyki częstotliwościowe (podstawowe)

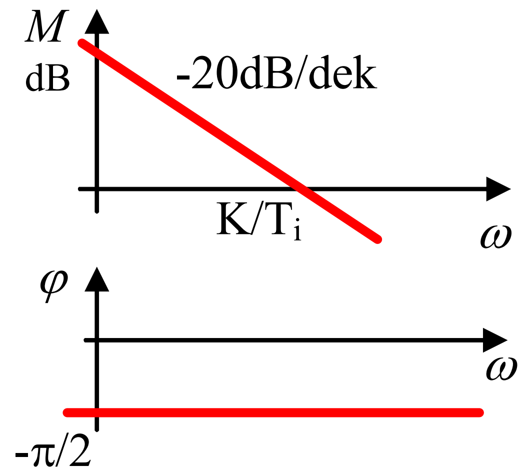
$$G(j\omega) = K$$



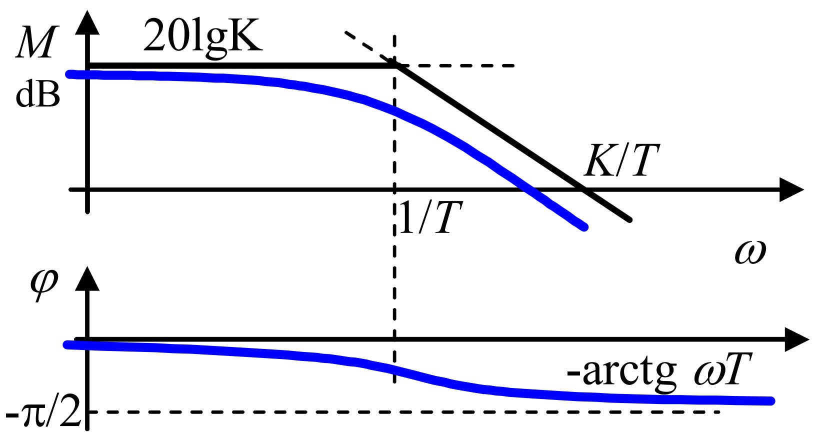

$$G(j\omega) = j\omega T_d$$



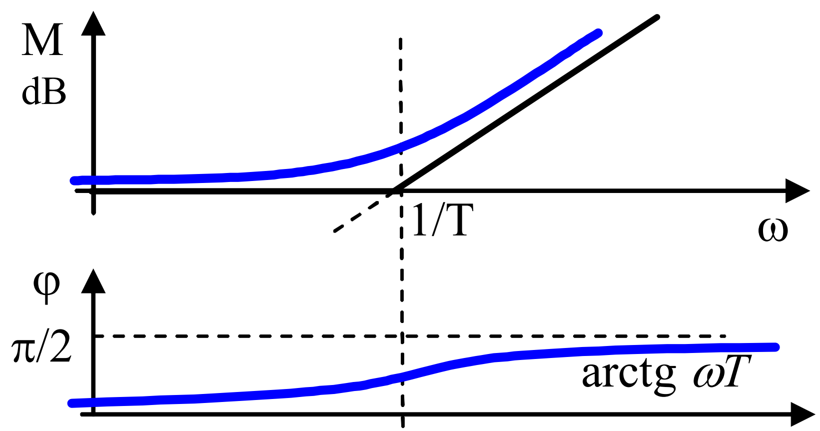

$$G(j\omega) = \frac{K}{j\omega T_i} = -j\frac{K}{\omega T_i}$$




$$G(j\omega) = \frac{K}{1 + j\omega T}$$

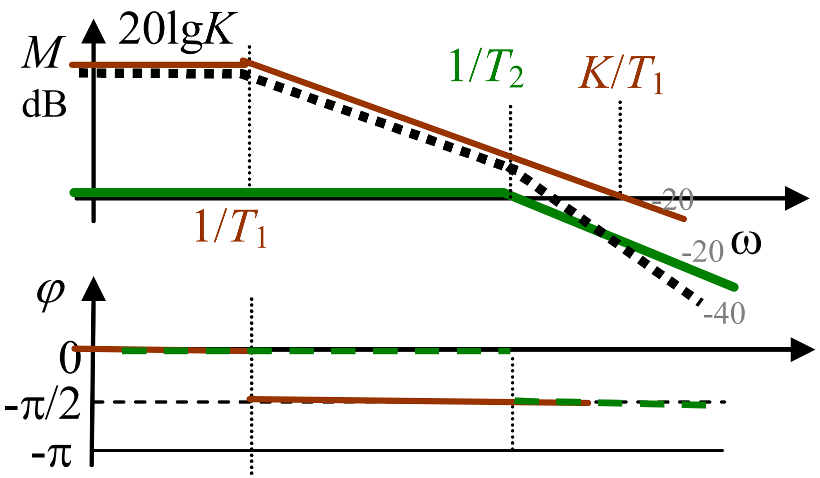


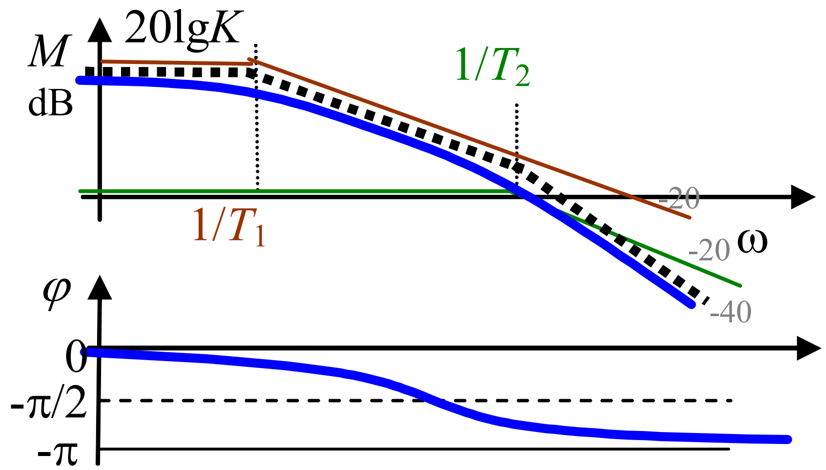

$$G(j\omega) = 1 + j\omega T$$



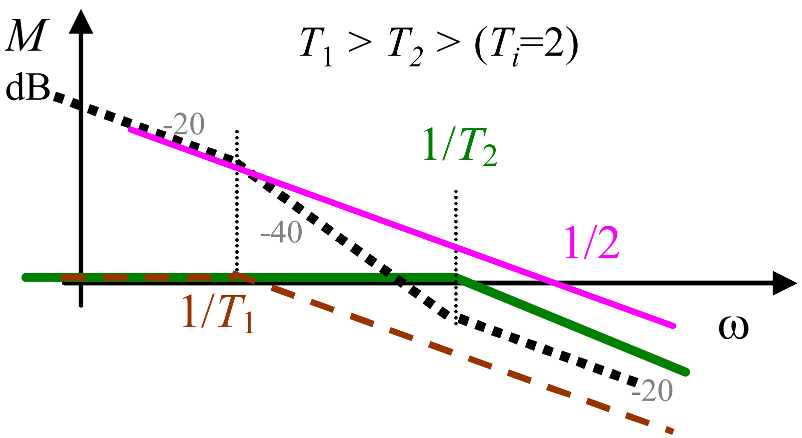
## Logarytmiczne charakterystyki częstotliwościowe (złożone)

$$\frac{K}{(1+j\omega T_1)(1+j\omega T_2)}, T_1 > T_2$$



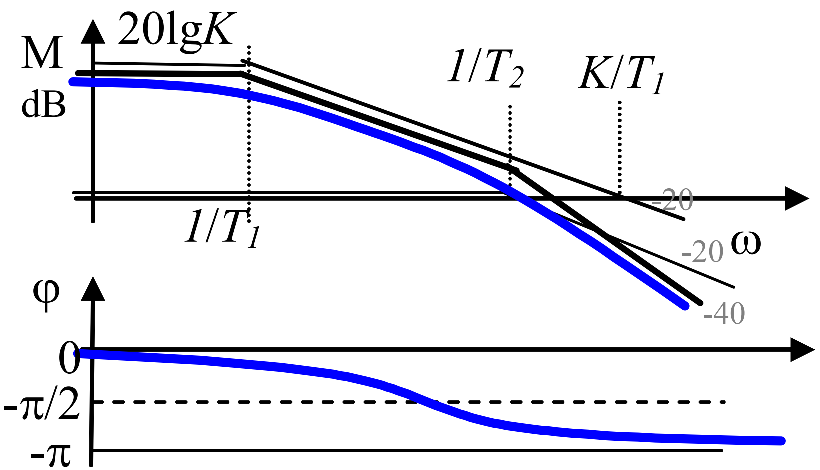




$$\frac{(1+j\omega T_2)}{2s(1+j\omega T_1)} = \frac{1}{2s}\frac{1}{(1+j\omega T_1)}(1+j\omega T_2)$$

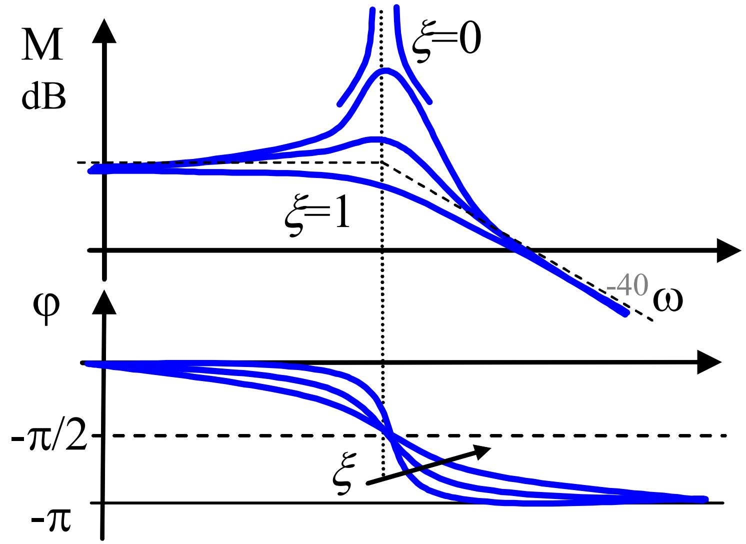


# Logarytmiczne charakterystyki częstotliwościowe (szczególne)

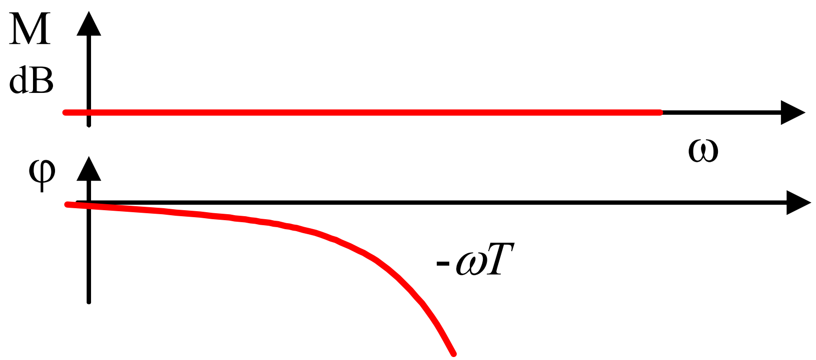
$$\frac{K}{1+j2\xi T\omega+(j\omega T)^2}=\frac{K}{(1+j\omega T_1)(1+j\omega T_2)},\xi>1$$



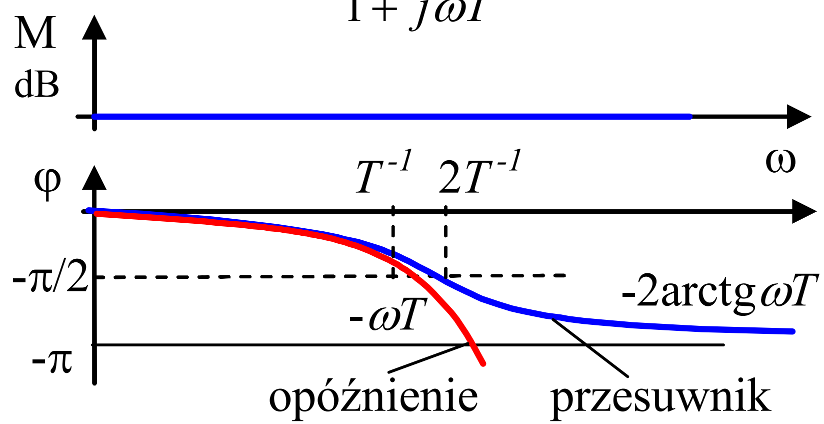

$$\frac{K}{1+j2\xi T\omega+(j\omega T)^2},\xi<1$$




$$G(j\omega)=e^{-j\omega T_o}$$

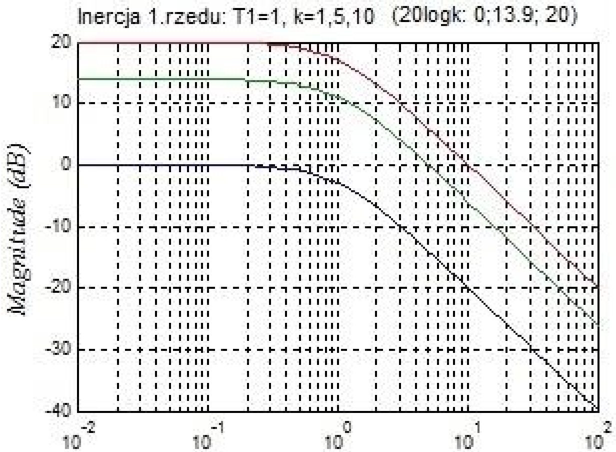



$$G(j\omega)=\frac{1-j\omega T}{1+j\omega T}$$

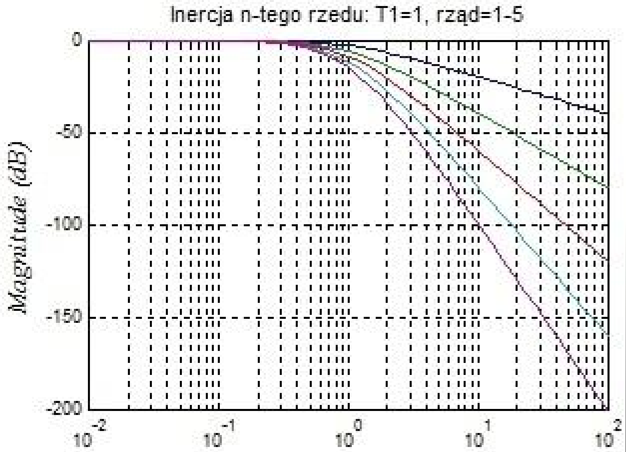


# Charakterystyki częstotliwościowe – wpływ parametrów

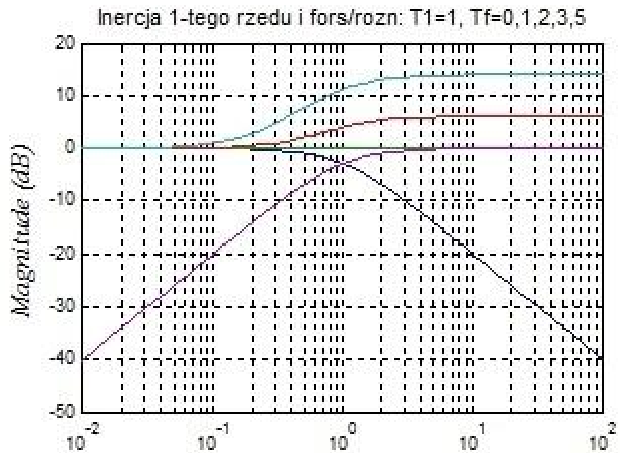
$$\frac{k}{(T_1 s+1)}$$

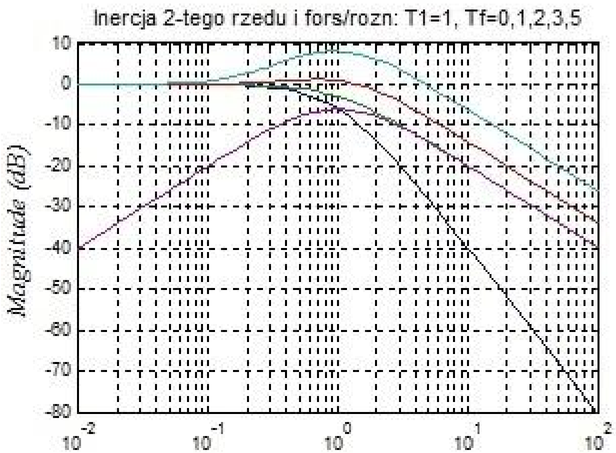



$$\frac{k}{(T_1 s+1)^n}$$

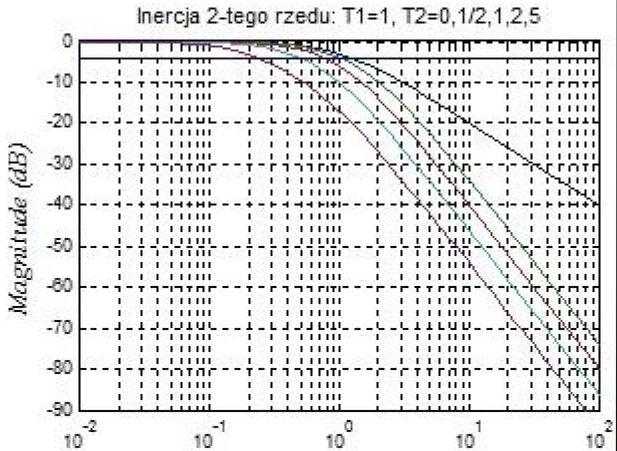



$$\frac{k(T_f s+1)}{(T_1 s+1)} \qquad \frac{ks}{(T_1 s+1)}$$




$$\frac{k(T_f s+1)}{(T_1 s+1)^2} \qquad \frac{ks}{(T_1 s+1)^2}$$




$$\frac{k}{(T_1 s+1)(T_2 s+1)}$$


$$\frac{k(T_f s+1)}{(T_1 s+1)(T_2 s+1)} \qquad \frac{ks}{(T_1 s+1)(T_2 s+1)}$$

# Regulator PID – charakterystyki częstotliwościowe

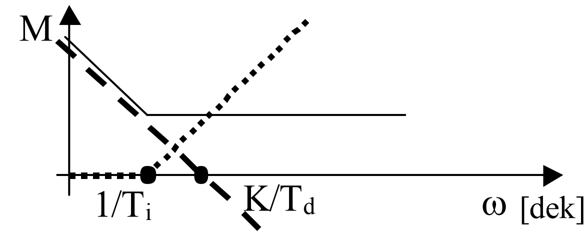
**PI:**

$$G(s) = K\left(1 + \frac{1}{sT_i}\right)$$

$$G(j\omega) = K - j\,\frac{K}{\omega T_i}$$

$$M(\omega) = 20\lg\left|K\,\frac{1 + j\omega T_i}{j\omega T}\right|$$

$$M(\omega) = 20\lg\left|\frac{K}{j\omega T}\right| + 20\lg\left|1 + j\omega T_i\right|$$





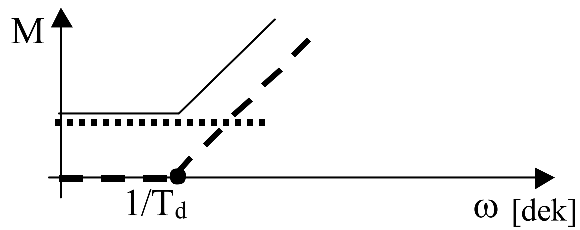
**PD:**

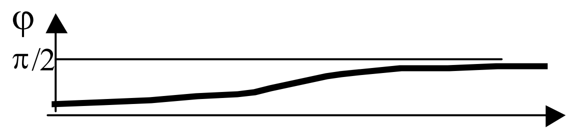
$$G(s) = K(1 + sT_d)$$

$$G(j\omega) = K + j\omega T_d$$

$$M(\omega) = 20\lg\left|K(1 + j\omega T_d)\right|$$

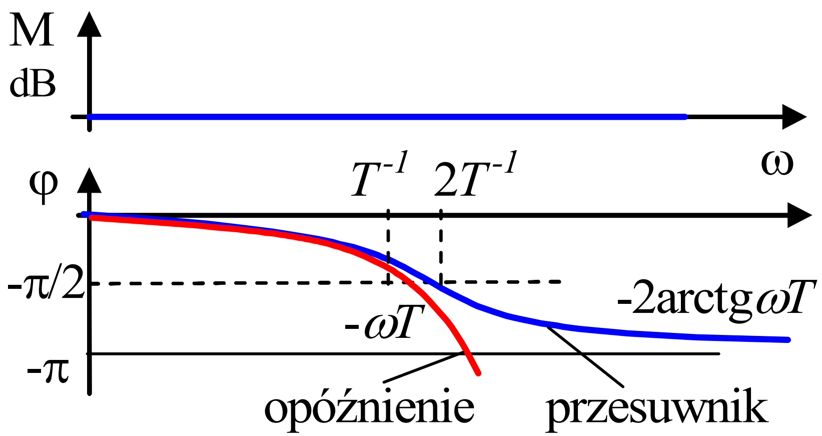
$$M(\omega) = 20\lg\left|K\right| + 20\lg\left|1 + j\omega T_d\right|$$



# Logarytmiczne charakterystyki częstotliwościowe

## układy minimalnofazowe



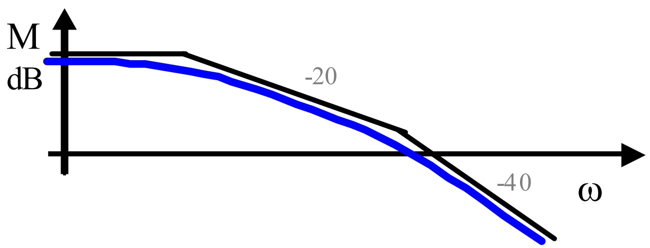
## Własności logarytmicznych charakterystyk częstotliwościowych

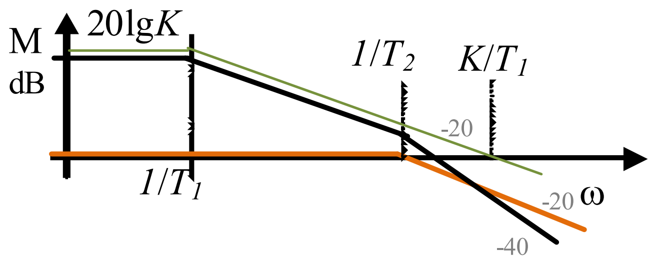
- sumowanie charakterystyk dla członów połączonych szeregowo
- asymptoty charakterystyki amplitudowej – nachylenie +/- 20 dB/dek
- każdy biegun objawia się załamaniem asymptoty o -20 dB/dek
- każde zero objawia się załamaniem asymptoty o +20 dB/dek
- określony maksymalny błąd charakterystyk asymptotycznych członu inercyjnego i forsującego
  - dla częstości załamania popełnia się błąd 3dB
  - w odległości oktawy od częstości załamania – błąd 1 dB
- dla układów minimalnofazowych można otworzyć ch. fazową

## Zdejmowanie charakterystyk częstotliwościowych

Opis eksperymentu

Identyfikacja modelu na podstawie charakterystyk częstotliwościowych

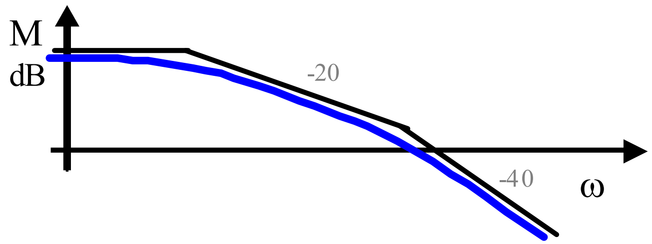


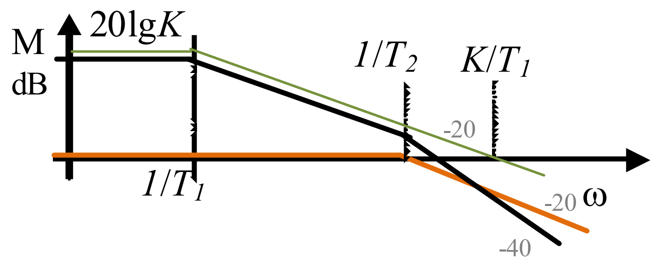




$$\frac{K}{(1+j\omega T_1)}\cdot\frac{1}{(1+j\omega T_2)}$$

# Zastosowanie charakterystyk częstotliwościowych

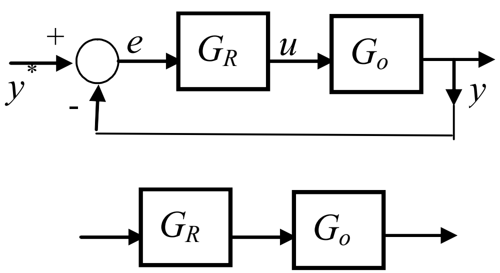
• Identyfikacja modelu na podstawie charakterystyk częstotliwościowych

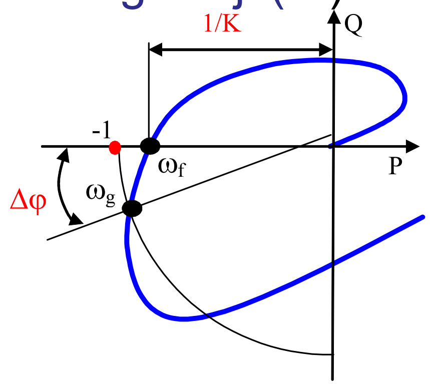


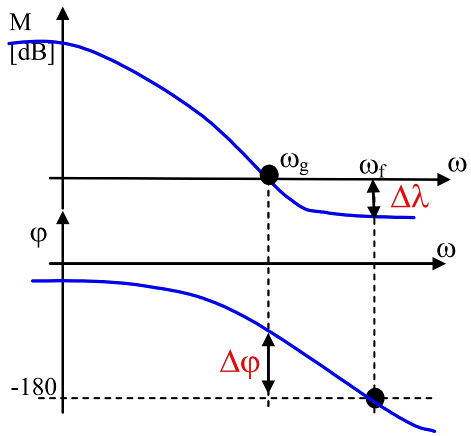



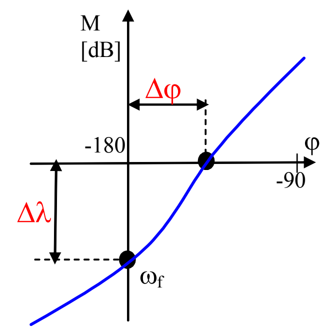

$$\frac{K}{(1+j\omega T_1)}\cdot\frac{1}{(1+j\omega T_2)}$$

• Kryterium stabilności układu regulacji (✵)




• Projektowanie filtrów (✵)

• Korekcja własności dynamicznych (✵)

  • pasmo przenoszenia

  • kompensacja biegunów