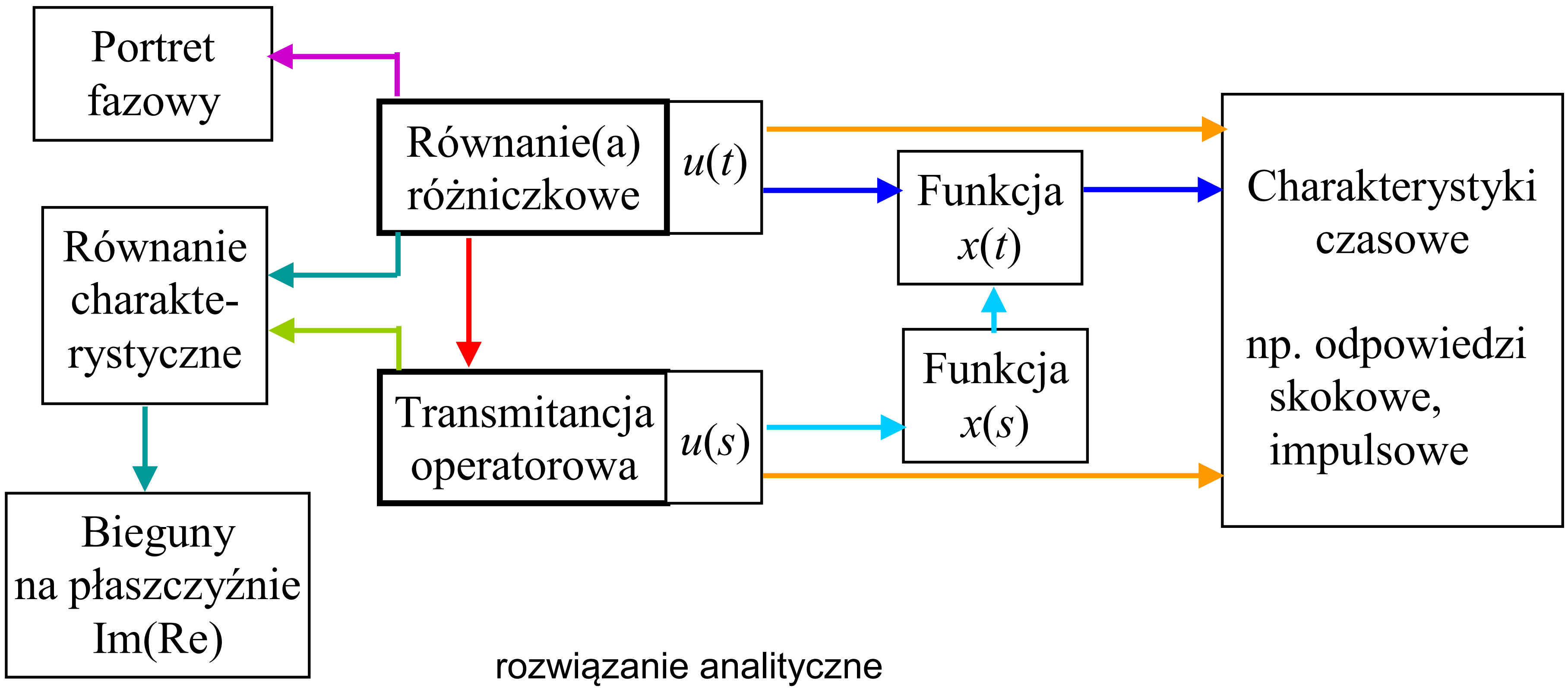
Opis własności dynamicznych (1)



rozwiązanie analityczne
charakterystyka czasowa
położenie biegunów
portret fazowy
przekształcenie Laplace'a,
    położenie biegunów
    transformata rozwiązania, przekształcenie odwrotne
rozwiązanie symulacyjne
charakterystyki częstotliwościowe

Układ równań stanu / transmitancja – własności obiektu

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -a_1 & a_2 \\ b_1 & -b_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$s^2 + (a_1 + b_2)s + a_1 b_2 - b_1 a_2 = 0$$

$$s\mathbf{x}(s) = \mathbf{A}\mathbf{x}(s) + \mathbf{B}\mathbf{u}(s)$$

$$\mathbf{x}(s) = \left(s\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B}\mathbf{u}(s)$$

$$\mathbf{x}(s) = \mathbf{G}(s)\mathbf{u}(s)$$

$$s^2 + (a_1 + b_2)s + a_1 b_2 - b_1 a_2 = 0$$

$$\begin{cases} (s + a_1)x_1(s) = a_2 x_2(s) + u_1(s) \\ (s + b_2)x_2(s) = b_1 x_1(s) + u_2(s) \end{cases}$$

$$\begin{cases} M_1(s)x_1(s) = a_2 x_2(s) + u_1(s) \\ M_2(s)x_2(s) = b_1 x_1(s) + u_2(s) \end{cases}$$

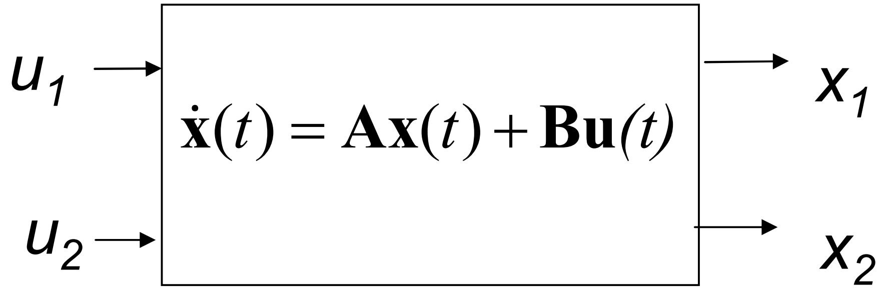
$$x_1(s) = \frac{M_2(s)}{M(s)}u_1(s) + \frac{a_2}{M(s)}u_2(s)$$

$$x_2(s) = \frac{b_1}{M(s)}u_1(s) + \frac{M_1(s)}{M(s)}u_2(s)$$

$$M(s) = M_1(s)M_2(s) - b_1 a_2$$

Układ równań stanu / transmitancja – „struktura” obiektu

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -a_1 & a_2 \\ b_1 & -b_2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$



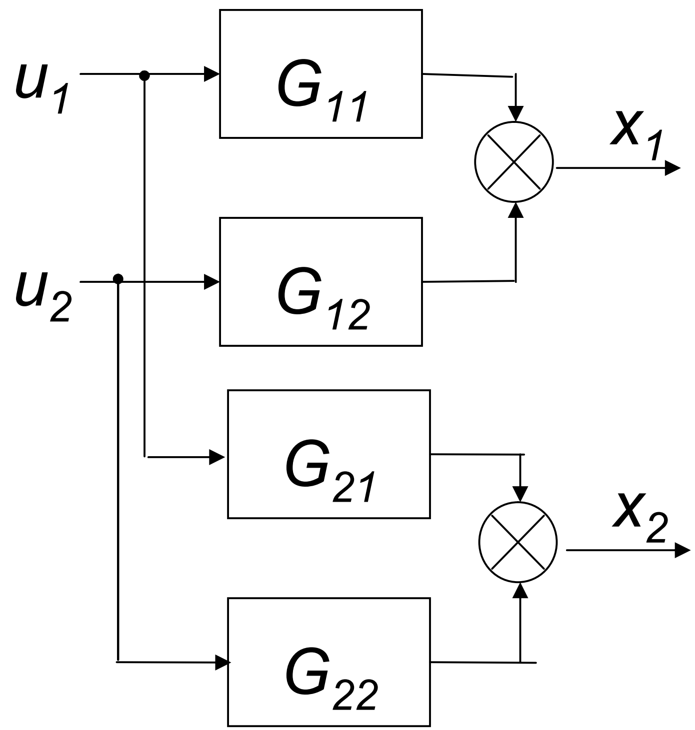
$$X_1(s) = \frac{s+b_2}{M(s)} U_1(s) + \frac{a_2}{M(s)} U_2(s)$$

$$X_2(s) = \frac{b_1}{M(s)} U_1(s) + \frac{s+a_1}{M(s)} U_2(s)$$

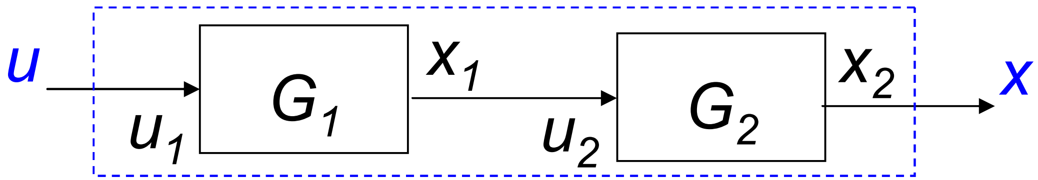
$$M(s) = s^2 + (a_1 + b_2)s + a_1 b_2 - b_1 a_2$$

$$X_1(s) = G_{11}(s)U_1(s) + G_{12}(s)U_2(s)$$

$$X_2(s) = G_{21}(s)U_1(s) + G_{22}(s)U_2(s)$$



Transmitancja - połączenia elementarne

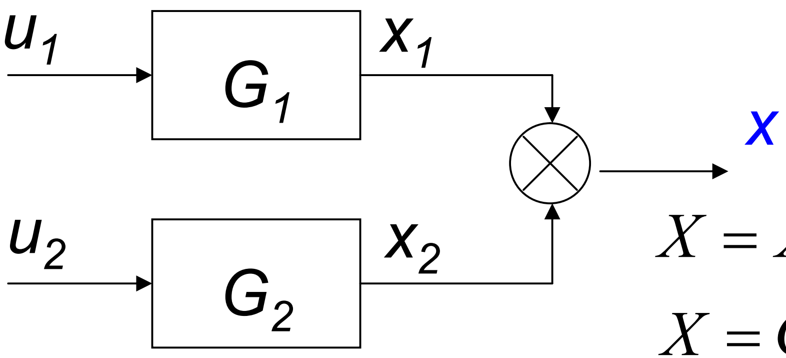


$$X_1 = G_1 U_1$$

$$X_2 = G_2 U_2 = G_2 X_1$$

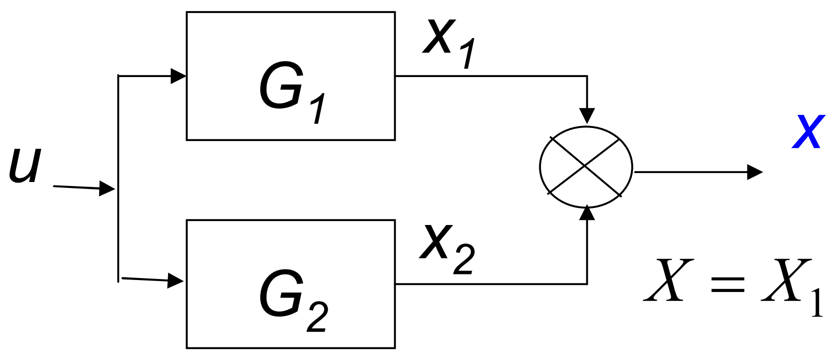
$$X_2 = G_1 G_2 U_1$$

$$X = G_1 G_2 U$$



$$X = X_1 + X_2$$

$$X = G_1 U_1 + G_2 U_2$$

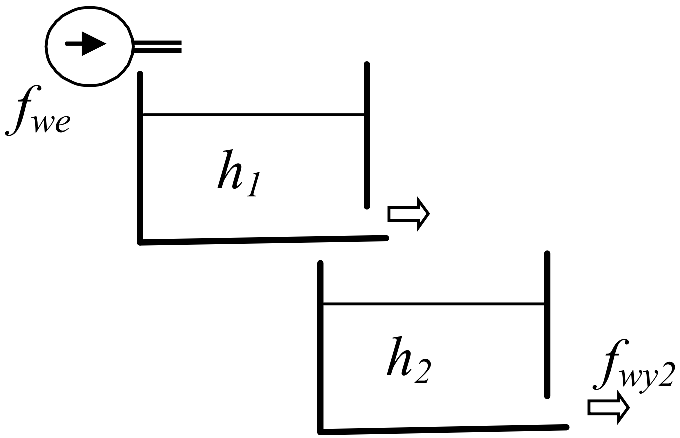


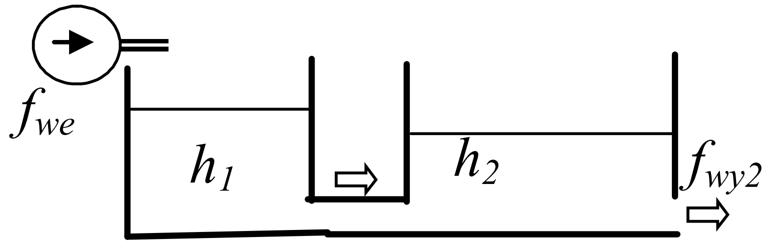
$$X = X_1 + X_2$$

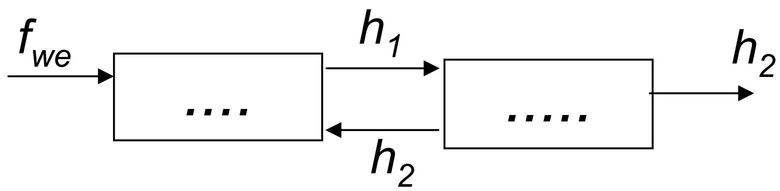
$$X = G_1 U + G_2 U$$

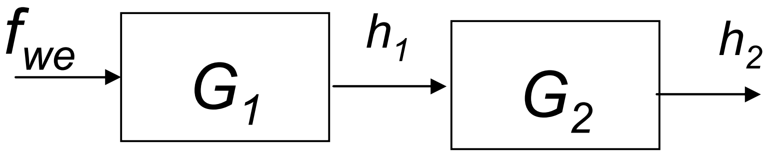
$$X = (G_1 + G_2) U$$

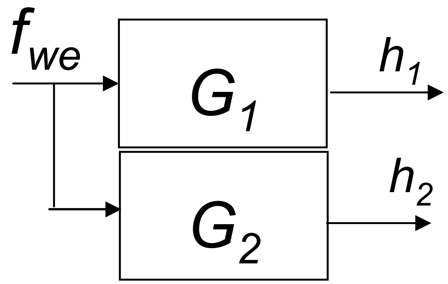
Kaskady niewspółdziałające i współdziałające











$$h_1(s) = \frac{\cdots}{M_1(s)} f_{we}(s)$$

$$h_2(s) = \frac{\cdots}{M_1(s) M_2(s)} f_{we}(s)$$

$$h_1(s) = \frac{\cdots}{M(s)} f_{we}(s)$$

$$h_2(s) = \frac{\cdots}{M(s)} f_{we}(s)$$

$$M(s) = M_1(s) M_2(s) + k$$

Człon inercyjny

$$G(s)=\frac{k}{Ts+1} \quad ,T>0$$

$$a_1\dot{x}(t)+a_0x(t)=b_0u(t)$$

$$G(s)=\frac{b_0}{a_1s+a_0}=\frac{b_0}{a_0\left(\frac{a_1}{a_0}s+1\right)} \qquad ,k=\frac{b_0}{a_0} \qquad ,T=\frac{a_1}{a_0}$$

---

r.s.: $a_0x=b_0u \rightarrow x=b_0u/a_0$ $\qquad \lim_{s\to 0} s\frac{k}{Ts+1}\frac{u_k}{s}=ku_k$ $\qquad , u(t)=u_k$

r.ch.: $a_1s+a_0=0 \rightarrow s_1=\frac{-a_0}{a_1}$ $\qquad Ts+1=0 \rightarrow s_1=\frac{-1}{T}$ $\qquad s_1<0 \rightarrow T>0$

Rozwiązanie ogólne dla u_k: $\quad x(t)=Ae^{(-1/T)t}+ku_k$

a) $x_s(t)=Ae^{s_1t}=Ae^{(-1/T)t}$

b) $x_w(t)=b_0u_k/a_0=ku_k$

Odpowiedź skokowa:

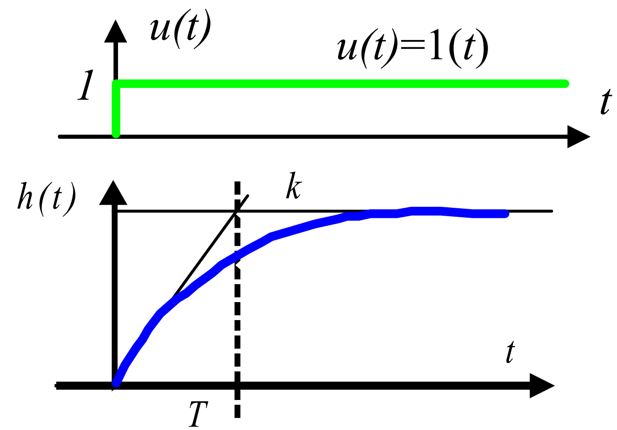
c) $u_k=1$

$$x(t)=-ke^{(-1/T)t}+k$$

$$x(t)=k\left(1-e^{(-1/T)t}\right)$$

d) $u(0)=0,\ x(0)=0$

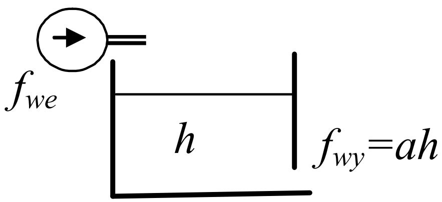
$0=Ae^{-1/T\cdot 0}+k \rightarrow A=-k$



Człon inercyjny - przykłady

$$G(s)=\frac{k}{Ts+1}\quad ,T>0$$

Obiekty
z samowyrównywaniem



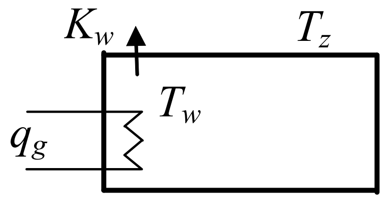
$$A\dot{h}(t)=f_{we}(t)-ah(t)$$

$$h(s)=\frac{1}{As+a}f_{we}(s)$$

$$k=\frac{1}{a}$$

$$T=\frac{A}{a}>0$$

Gdy $A\uparrow$, to …



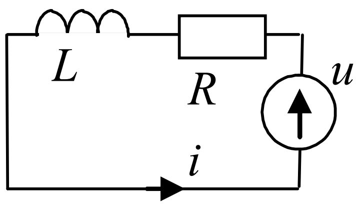
$$C_V\dot{T}_w(t)=q_g(t)-K_{cw}\left(T_w(t)-T_z(t)\right)$$

$$T_w(s)=\frac{1}{C_Vs+K_{cw}}q_g(s)+\frac{K_{cw}}{C_Vs+K_{cw}}T_z(s)$$

$$k=\frac{1}{K_{cw}}\qquad k=1$$

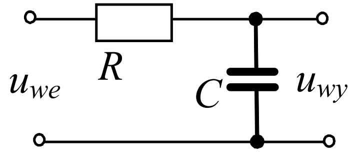
$$T=\frac{C_v}{K_{cw}}>0$$

Gdy $K_{cw}\uparrow$, to …
Gdy $V\uparrow$, to …



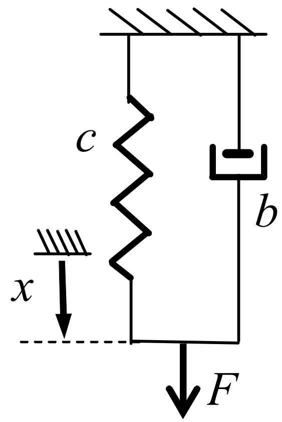
$$sLi(s)+Ri(s)=u(s)$$

$$i(s)=\frac{1}{sL+R}u(s)$$



$$u_{wy}(s)=\frac{1}{sC}\frac{u_{we}(s)}{R+1/(sC)}$$

$$u_{wy}(s)=\left(\frac{1}{sCR+1}\right)u_{we}(s)$$



$$b\dot{x}(t)+cx(t)=F(t)$$

$$x(s)=\frac{1}{sb+c}F(s)$$

Człon całkujący

$$a_1\dot{x}(t)=b_0u(t)$$

$$G(s)=\frac{b_0}{a_1 s} \qquad ,T_i=\frac{a_1}{b_0}$$

$$G(s)=\frac{1}{T_i s}=k_i\frac{1}{s}$$

r.s.: $\lim\limits_{s\to 0} s\frac{1}{T_i s}\frac{u_k}{s}=\infty$, $u(t)=u_k$

$\lim\limits_{s\to 0} s\frac{1}{T_i s}1=\frac{1}{T_i}$, $u(t)=\delta(t)$

r.ch.: $a_1 s=0 \;\rightarrow\; s=0$

Rozwiązanie ogólne dla u_k: $x(t)=k_iu_kt+x_0$

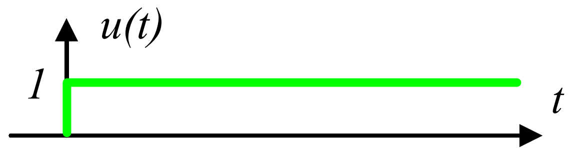
a) $x(t)=k_i\int u_k dt+x_0$

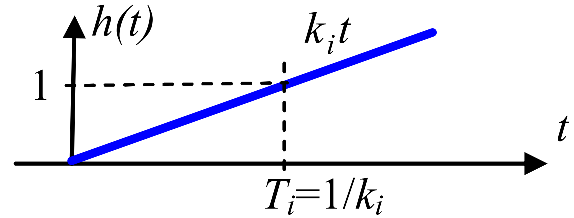
Odpowiedź skokowa: $x(t)=k_it$

b) $u_k=1$

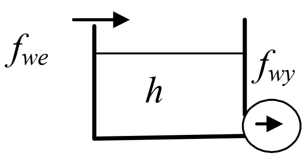
c) $u(0)=0$, $x(0)=0$

$0=k_it+x_k$



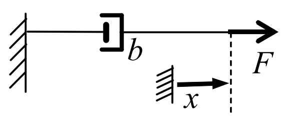


Człon całkujący - przykłady



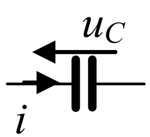
$$A\dot{h}(t)=f_{we}(t)-f_{wy}(t)$$

$$h_1(s)=\frac{1}{As}f_{we}(s)-\frac{1}{As}f_{wy}(s)$$



$$b\dot{x}(t)=F(t)$$

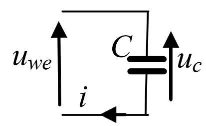
$$x(s)=\frac{1}{sb}F(s)$$



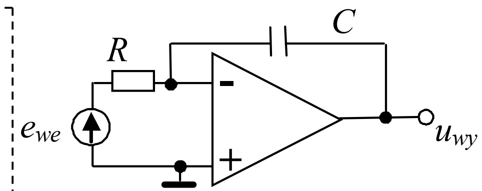
$$u_C(t)=\frac{1}{C}\int i(t)dt$$

$$u_C(s)=\frac{1}{sC}i(s)$$

$$\left(u_C(t)=\frac{1}{C}q(t)\right)$$



$$u_c(s)=\frac{1}{sC}i(s)$$



$$u_{wy}=\frac{-1}{sRC}e_{we}$$

Człon oscylacyjny

$$a_2\ddot{x}(t)+a_1\dot{x}(t)+a_0x(t)=b_0u(t)$$

$$G(s)=\frac{k1}{s^2+2\,\xi\,\omega_n s+\omega_n^2}\quad,\omega_n>0$$

$$G(s)=\frac{k}{T_n^2 s^2+2\,\xi\,T_n\,s+1}\quad,T_n>0$$

$$G(s)=\frac{b_0}{a_2s^2+a_1s+a_0}=\frac{b_0}{a_2\left(s^2+\frac{a_1}{a_2}s+\frac{a_0}{a_2}\right)}=\frac{b_0}{a_0\left(\frac{a_2}{a_0}s^2+\frac{a_1}{a_0}s+1\right)}$$

Jeśli $a_0/a_2>0$, to: $\omega_n^2=\frac{a_0}{a_2}$, $2\xi\omega_n=\frac{a_1}{a_2}$, $k_1=\frac{b_0}{a_2}$ $\qquad T^2=\frac{a_2}{a_0}$, $2\xi T=\frac{a_1}{a_0}$, $k=\frac{b_0}{a_0}$

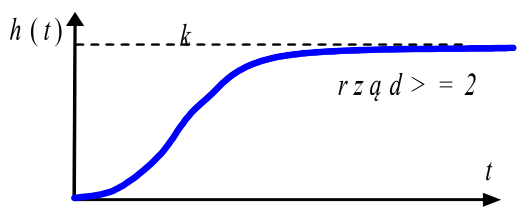
---

r.s.: $a_0x=b_0u$ $\qquad \lim_{s\to 0} s\frac{k_1}{s^2+2\xi\omega_n s+\omega_n^2}\frac{u_0}{s}=\frac{k_1u_0}{\omega_n^2}$, $u(t)=u_0$

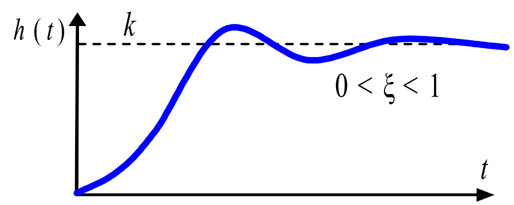
r.ch.: $a_2s^2+a_1s+a_0=0\to s_1,s_2$ $\qquad s^2+2\xi\omega_n s+\omega_n^2=0$ $\qquad T^2s^2+2\xi Ts+1=0$

$\mathrm{Im}(s_i)=0,\ \mathrm{Re}(s_i)<0$

$$\frac{k}{(T_1s+1)(T_2s+1)}\longleftarrow G(s)=\frac{b_0}{a_2(s-s_1)(s-s_2)}\longrightarrow\frac{k_1}{s^2+2\,\xi\,\omega\,s+\omega^2}$$



Człon inercyjny
II rzędu



Człon oscylacyjny znormalizowany

$$\frac{\omega_n^2}{s^2 + 2\,\xi\,\omega_n s + \omega_n^2}\;,\omega_n > 0 \qquad\qquad \frac{1}{T_n^2 s^2 + 2\,\xi\,T_n\,s + 1}\;,T_n > 0$$

$$\frac{\sigma^2 + \omega_r^2}{s^2 + 2\,\sigma s + \sigma^2 + \omega_r^2}$$

$$s_1 = -\xi\omega_n + \omega_n\sqrt{\xi^2 - 1}$$

$$s_2 = -\xi\omega_n - \omega_n\sqrt{\xi^2 - 1}$$

Gdy

$$\xi^2 - 1 < 0$$

$$s_1 = \alpha + j\omega_r$$

$$s_2 = \alpha - j\omega_r$$

gdzie:

$$\alpha = -\xi\omega_n$$

$$\omega_r = \omega_n\sqrt{1 - \xi^2}$$

$$s_1 = -\sigma + j\omega_r$$

$$s_2 = -\sigma - j\omega_r$$

gdzie:

$$\sigma = \xi\omega_n$$

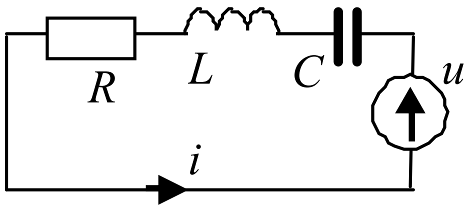
$$\omega_r = \omega_n\sqrt{1 - \xi^2}$$

$$\frac{\sigma^2 + \omega_r^2}{s^2 + 2\,\sigma s + \sigma^2 + \omega_r^2} = \frac{(\xi\omega_n)^2 + \left(\omega_n\sqrt{1 - \xi^2}\right)^2}{s^2 + 2\,\xi\omega_n s + (\xi\omega_n)^2 + \left(\omega_n\sqrt{1 - \xi^2}\right)^2} = \frac{\omega_n^2}{s^2 + 2\,\xi\omega_n s + \omega_n^2}$$

Człon oscylacyjny - przykłady

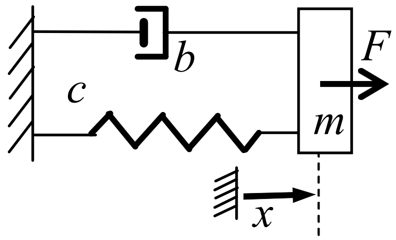
$$G(s) = \frac{k1}{s^2 + 2\,\xi\,\omega_n s + \omega_n^2} \quad ,\omega_n > 0$$

$$G(s) = \frac{k}{T_n^2 s^2 + 2\,\xi\,T_n\; s + 1} \quad ,T_n > 0$$



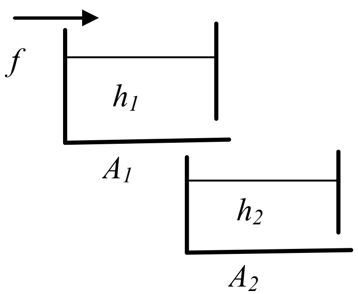
$$sLi(s) + Ri(s) + \frac{1}{sC} i(s) = u(s)$$

$$\frac{i(s)}{u(s)} = \frac{sC}{s^2 LC + sRC + 1}$$



$$m\ddot{x}(t) + b\dot{x}(t) + cx(t) = F(t)$$

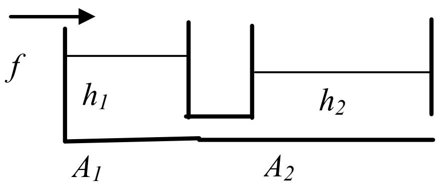
$$\frac{x(s)}{F(s)} = \frac{1}{ms^2 + bs + c}$$



$$\begin{cases} A_1\dot{h}_1(t) = f(t) - a_1 h_1(t) \\ A_2\dot{h}_2(t) = a_1 h_1(t) - a_2 h_2(t) \end{cases}$$

$$\frac{h_2(s)}{f(s)} = \frac{1}{(A_1 s + a_1)(A_2 s + a_2)}$$

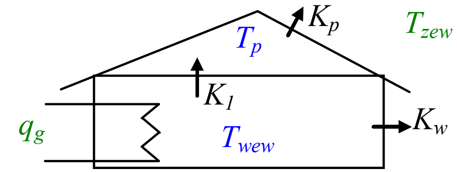
$$\xi > 1$$



$$\begin{cases} A_1\dot{h}_1(t) = f(t) - a_1\big(h_1(t) - h_2(t)\big) \\ A_2\dot{h}_2(t) = a_1\big(h_1(t) - h_2(t)\big) - a_2 h_2(t) \end{cases}$$

$$\frac{h_2(s)}{f(s)} = \frac{a_1}{(A_1 s + a_1)(A_2 s + a_1 + a_2) - a_1^2}$$

$$\xi = ?$$



$$\xi = ?$$

Człon proporcjonalny

$$a_0 x(t) = b_0 u(t)$$

$$G(s) = k \qquad , k = \frac{b_0}{a_0}$$

$$\boxed{G(s) = k}$$

---

r.s.: $a_0 x = b_0 u$

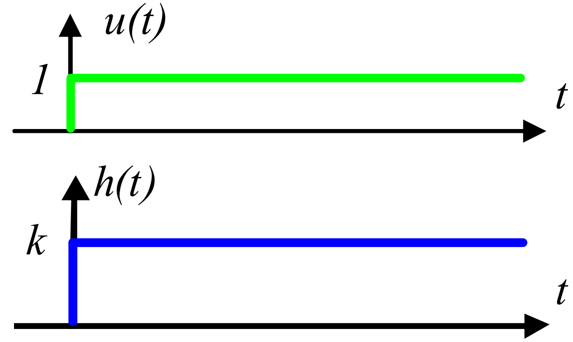
r.ch.:

Odpowiedź skokowa:

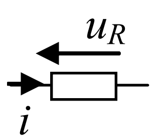
a) $u_k = 1$

b) $u(0)=0$, $x(0)=0$

$$\boxed{x(t) = k1(t)}$$

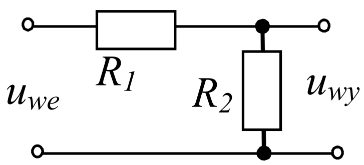


Człon proporcjonalny - przykłady



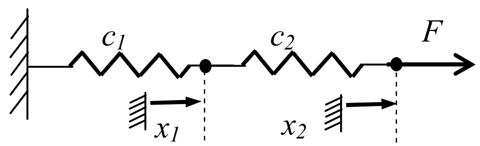
$$u_R(t) = Ri(t)$$

$$\frac{u_R(s)}{i(s)} = R$$



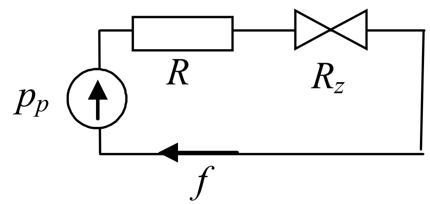
$$u_{wy}(t) = \left(\frac{R_2}{R_1 + R_2}\right) u_{we}(t)$$

$$\frac{u_{wy}(s)}{u_{we}(s)} = \frac{R_2}{R_1 + R_2}$$



$$\begin{cases} 0 = c_1 x_1(t) + c_2\left(x_1(t) - x_2(t)\right) \\ F(t) = c_2\left(x_2(t) - x_1(t)\right) \end{cases}$$

$$x_1 = \frac{F}{c_1} \qquad x_2 = \frac{c_1 + c_2}{c_1 c_2} F$$



$$p_p(t) = (R + R_z) f(t)$$

$$f = \frac{1}{R + R_z} p_p$$

Człon różniczkujący

$$a_0 x(t) = b_1 \dot{u}(t)$$

$$G(s) = T_d s \qquad , T_d = \frac{b_1}{a_0}$$

$$G(s) = T_d s$$

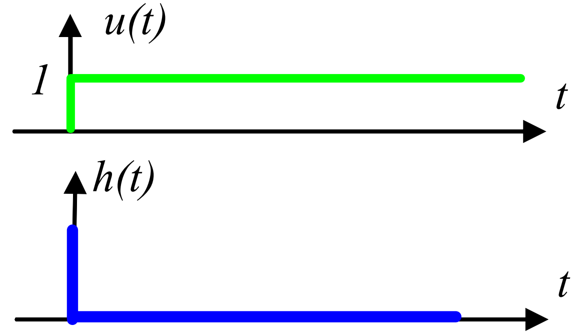
r.s.: $a_0 x = 0$

r.ch.:

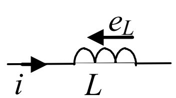
Odpowiedź skokowa:
a) $u_k = 1$

b) $u(0) = 0$, $x(0) = 0$

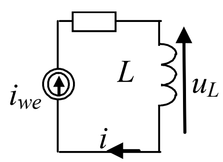
$$x(t) = T_d \delta(t)$$



Człon różniczkujący - przykłady

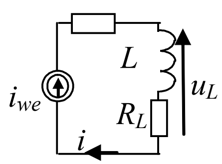


$$e_L(t) = -L\frac{di(t)}{dt}$$



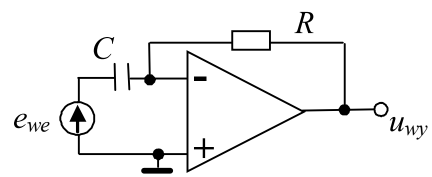
$$u_L(t) = L\frac{di(t)}{dt}$$

$$u_L(s) = sLi(s)$$



$$u_L(t) = L\frac{di(t)}{dt} + R_L i(t)$$

$$u_L(s) = (sL + R_L)i(s)$$



$$u_{wy} = -sRCe_{we}$$

Podstawowe obiekty (człony) dynamiki

$a_0 x(t) = b_0 u(t)$

$$G(s) = k$$

$a_1 \dot{x}(t) + a_0 x(t) = b_0 u(t)$

$$G(s) = \frac{k}{Ts+1}, T > 0$$

$\ddot{x}(t) + 2\xi\omega\dot{x}(t) + \omega^2 x(t) = b_0 u(t)$

$$G(s) = \frac{k1}{s^2 + 2\,\xi\,\omega_n\,s + \omega_n^2}, \omega_n > 0$$

$$G(s) = \frac{k}{T_n^2 s^2 + 2\,\xi\,T_n\,s + 1}, T_n > 0$$

$a_1 \dot{x}(t) = b_0 u(t)$

$$G(s) = \frac{1}{T_i s}$$

$a_0 x(t) = b_1 \dot{u}(t)$

$$G(s) = T_d s$$

k, k_1 – współczynniki wzmocnienia członu
T – stała czasowa
T_o – opóźnienie
T_i – czas całkowania
T_d – czas różniczkowania
ω_n – pulsacja drgań własnych niełtumionych
T_n, – okres drgań własnych niełtumionych
(współczynnik okresu drgań własnych)

$\omega_n = \frac{1}{T_n}$ $\qquad \omega_n = 2\pi f = \frac{2\pi}{T}$

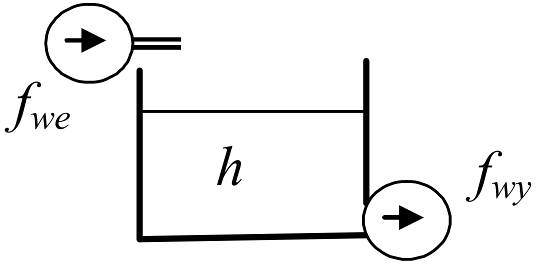
ξ – współczynnik tłumienia względnego

$$G(s) = \frac{L(s)}{M(s)}$$

$x(t) = u(t - T_0)$

$$G(s) = e^{-T_0 s}$$

Przykłady obiektów bez/z samowyrównywaniaem

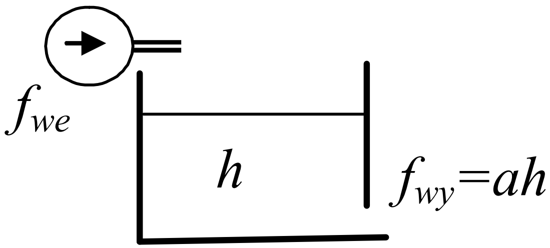


$$A\dot{h}(t) = f_{we}(t) - f_{wy}(t)$$

$$h_1(s) = \frac{1}{As} f_{we}(s) - \frac{1}{As} f_{wy}(s)$$

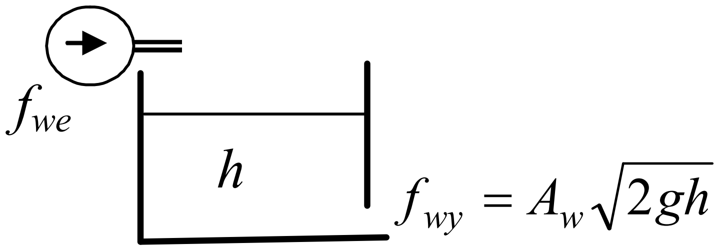
---

samowyrównywanie

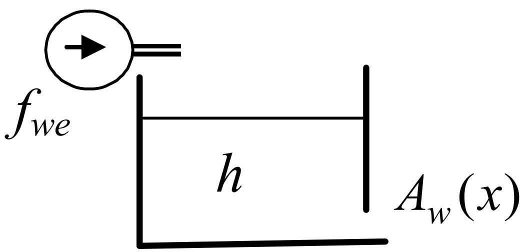


$$A\dot{h}(t) = f_{we}(t) - ah(t)$$

$$h(s) = \frac{1}{As + a} f_{we}(s)$$

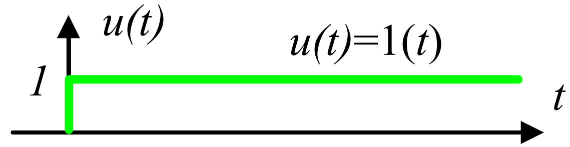


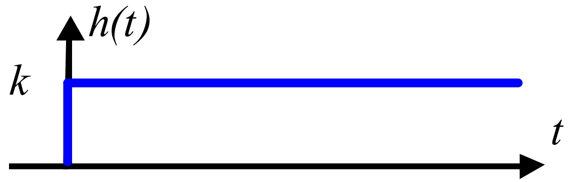
$$A\dot{h}(t) = f_{we}(t) - A_w\sqrt{2gh(t)}$$



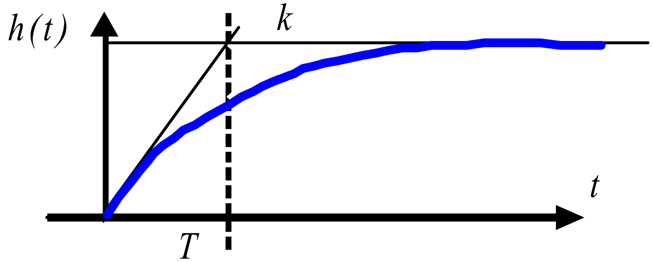
$$A\dot{h}(t) = f_{we}(t) - A_w\big(x(t)\big)\sqrt{2gh(t)}$$

Odpowiedzi skokowe członów podstawowych

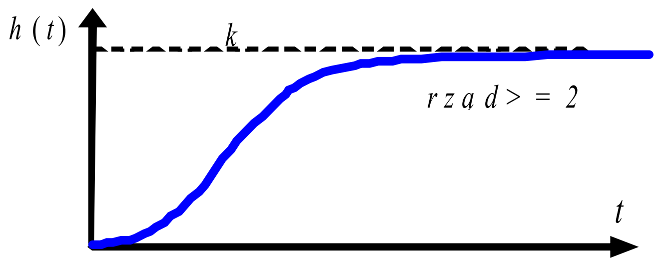




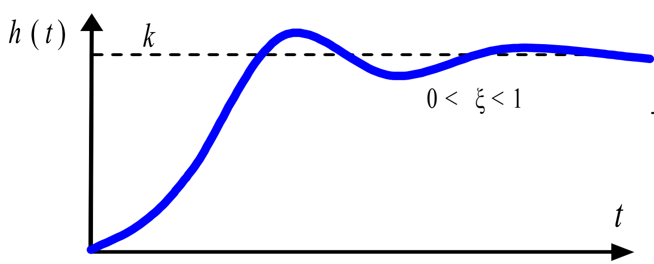
$$k$$



$$\frac{k}{Ts+1}$$

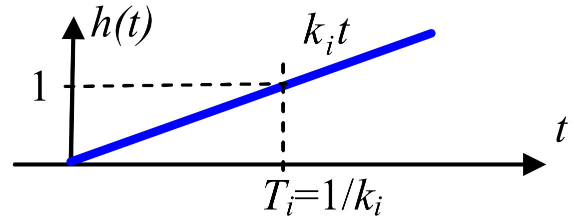


$$\frac{k}{(T_1 s+1)(T_2 s+1)}$$



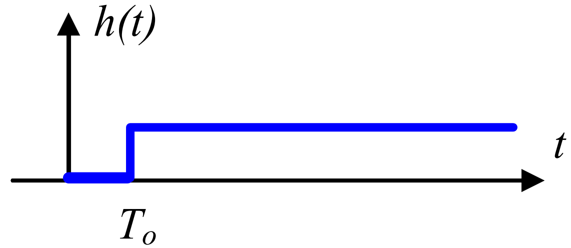
$$\frac{k}{s^2+2\,\xi\,\omega_n\,s+\omega_n^2}$$

$$\frac{K}{T_n^2 s^2+2\,\xi T_n\,s+1}$$



$$\frac{1}{T_i s}=\frac{k_i}{s}$$

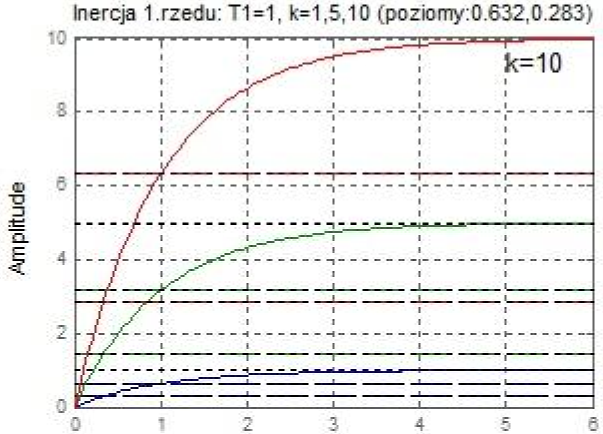
$$T_d s$$



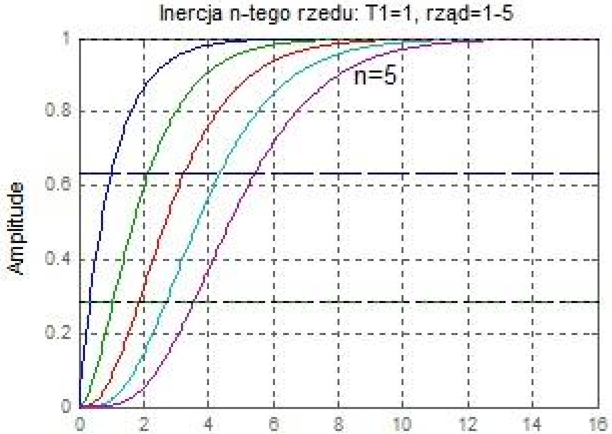
$$e^{-T_o s}$$

Odpowiedzi skokowe – wpływ parametrów

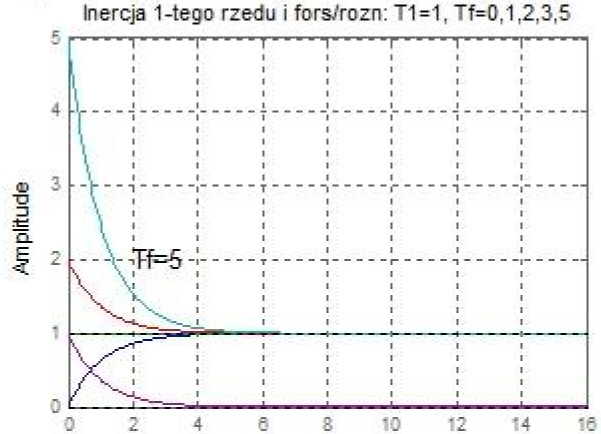
$$\frac{k}{(T_1 s+1)}$$



$$\frac{k}{(T_1 s+1)^n}$$



$$\frac{k(T_f s+1)}{(T_1 s+1)} \qquad \frac{ks}{(T_1 s+1)}$$

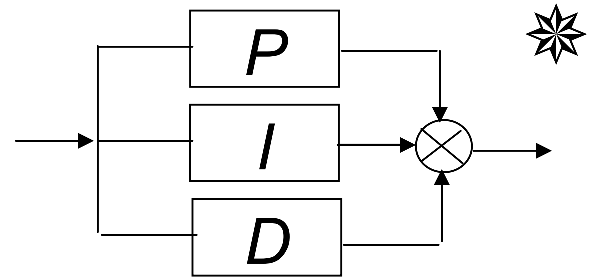


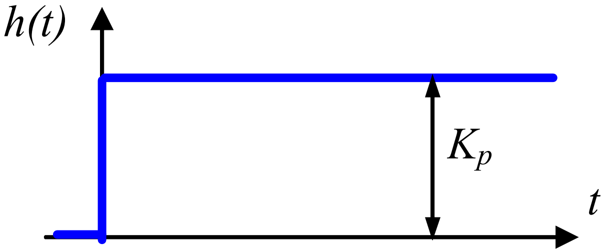
$$\frac{k(T_f s+1)}{(T_1 s+1)^2} \qquad \frac{ks}{(T_1 s+1)^2}$$

$$\frac{k}{(T_1 s+1)(T_2 s+1)}$$

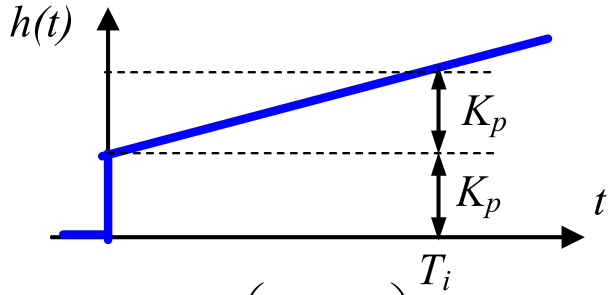
$$\frac{k(T_f s+1)}{(T_1 s+1)(T_2 s+1)} \qquad \frac{ks}{(T_1 s+1)(T_2 s+1)}$$

Regulator PID – odpowiedzi skokowe

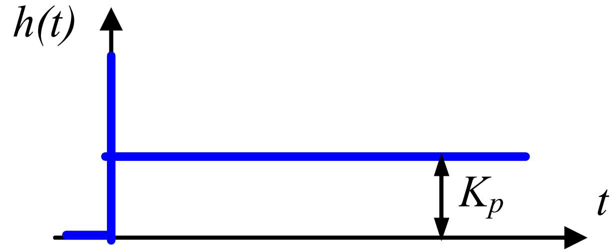




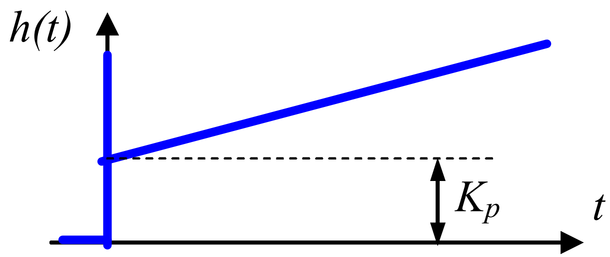
$$\mathrm{P}: K_p$$



$$\mathrm{PI}: K_p\left(1+\frac{1}{T_i s}\right)$$

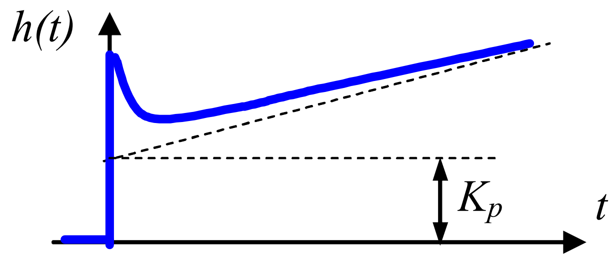


$$\mathrm{PD}: K_p\left(1+T_d s\right)$$



$$\mathrm{PID}: K_p\left(1+\frac{1}{T_i s}+T_d s\right)$$

idealny



$$\mathrm{PID}: K_p\left(1+\frac{1}{T_i s}+\frac{T_d s}{Ts+1}\right)$$

rzeczywisty

Człony o zadanych parametrach

1) Cz.inercyjny z biegunem s_1:

$$G(s)=\frac{a}{s-s_1}=\frac{a}{-s_1\left(\frac{1}{-s_1}s+1\right)}=\frac{k}{Ts+1}$$

a) wzmocnienie członu inercyjnego = 1

$$k=1 \quad \rightarrow \frac{a}{-s_1}=1 \ \rightarrow a=-s_1$$

b) wzmocnienie układu K_0:

$$\lim_{s\to 0} sG(s)\frac{1}{s}=K_0 \quad \rightarrow \quad \frac{a}{-s_1}=K_0 \rightarrow a=\ldots$$

---

2) Cz.oscylacyjny o tłumieniu* ½ i pulsacji** 2:

$$G(s)=\frac{a}{s^2+2\xi\omega s+\omega^2}=\frac{a}{s^2+2s+4}$$

$$=\frac{a}{(s-s_1)(s-s_2)}=\frac{b}{(T_1s+1)(T_2s+1)}$$

a) wzmocnienie członu oscylacyjnego = 1

$$a=1$$

b) wzmocnienie układu K_0:

$$\lim_{s\to 0} sG(s)\frac{1}{s}=K_0 \quad \rightarrow \quad \frac{a}{\omega^2}=K_0 \rightarrow a=\ldots$$

---

3) Cz.oscylacyjny o tłumieniu ½ i okresie** 2:

$$G(s)=\frac{a}{T^2s^2+2\xi Ts+1}=\frac{a}{4s^2+2s+1}$$

a) wzmocnienie członu oscylacyjnego = 1

$$a=1$$

b) wzmocnienie układu K_0:

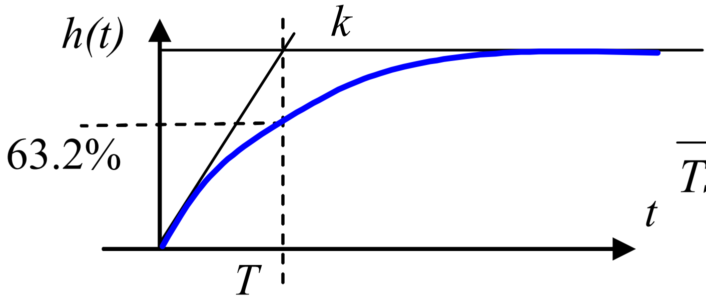
$$\lim_{s\to 0} sG(s)\frac{1}{s}=K_0 \quad \rightarrow \quad a=K_0$$

---

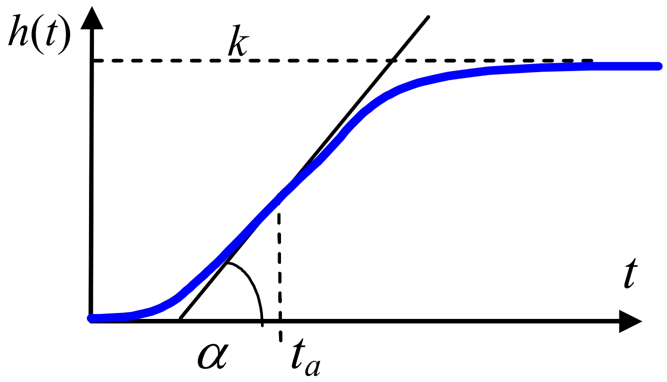
* tłumienie - współczynnik tłumienia względnego

** pulsacja/okres – pulsacja/okres drgań własnych nietłumionych

Identyfikacja modelu na podstawie odpowiedzi na wymuszenie skokowe

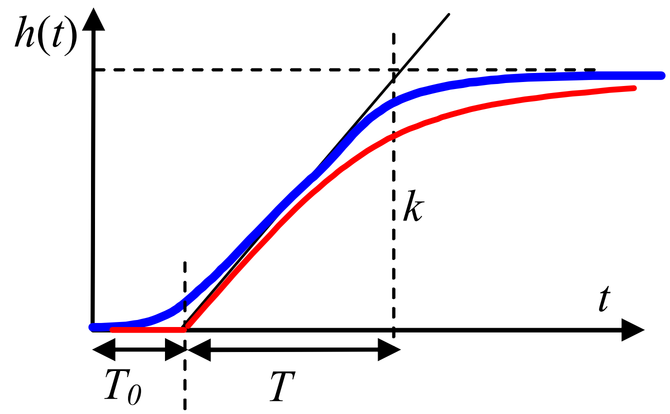


$$\frac{k}{Ts+1}, \quad k = \frac{\Delta x}{\Delta u}$$

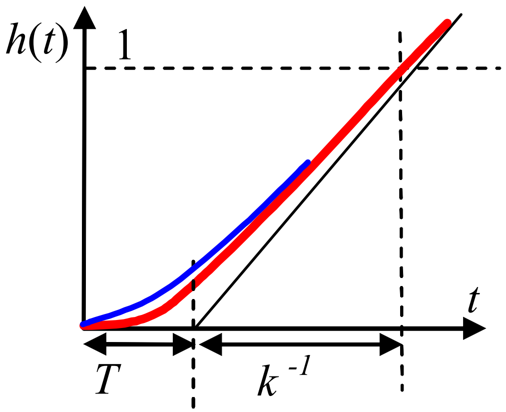


$$\frac{k}{(1+sT)^n}$$

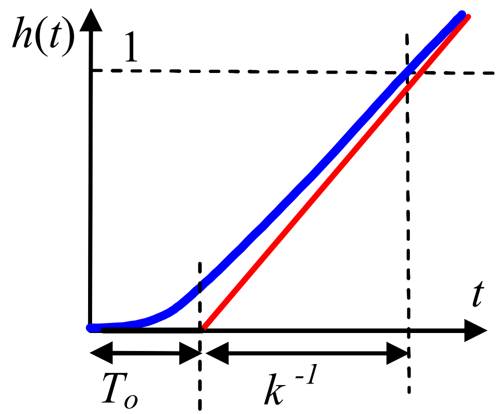
$$\frac{k}{(1+sT)^n} e^{-sT_o}$$



$$\frac{k}{Ts+1} e^{-sT_o}$$



$$\frac{k}{s(Ts+1)}$$



$$\frac{k}{s} e^{-sT_o}$$