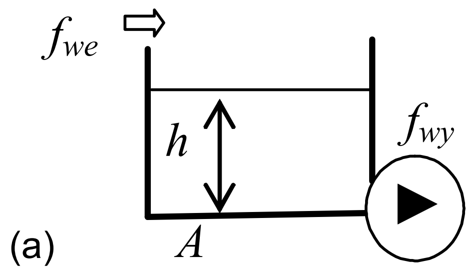
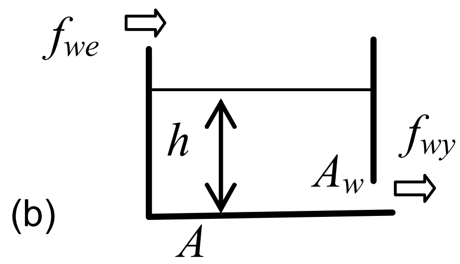
Otwarte układy hydrauliczne





1) Zawartość magazynu $V(t) = Ah(t)$

2) Zmiana zawartości magazynu $$\frac{dV(t)}{dt} = A\frac{dh(t)}{dt} = A\dot{h}(t)$$

3) Bilans strumieni wpływających i wypływających [m3/s] $$A\dot{h}(t) = f_{we}(t) - f_{wy}(t)$$

(a) $f_{wy}(t)$

(a) $$A\dot{h}(t) = f_{we}(t) - f_{wy}(t)$$

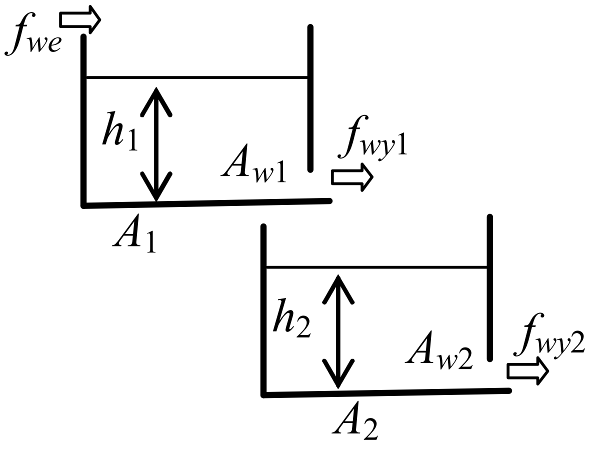
(b) $$f_{wy}(t) = A_w\sqrt{2gh(t)} \approx ah(t)$$

(b_1) $$A\dot{h}(t) = f_{we}(t) - A_w\sqrt{2gh(t)}$$

(b_2
) $$A\dot{h}(t) = f_{we}(t) - ah(t)$$

4) Zmienne wejściowe i wyjściowe, kompletność modelu

Otwarte układy hydrauliczne



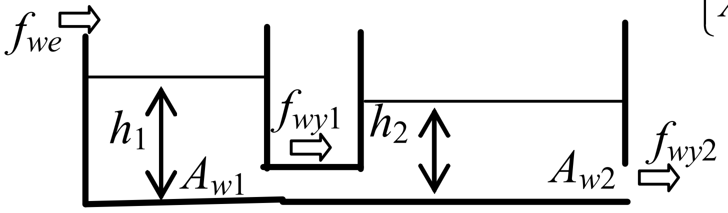
$$\begin{cases} A_1\dot{h}_1(t) = f_{we}(t) - f_{wy1}(t) \\ A_2\dot{h}_2(t) = f_{wy1}(t) - f_{wy2}(t) \end{cases}$$

$$f_{wy1}(t) = A_{w1}\sqrt{2gh_1(t)} \approx a_1h_1(t)$$

$$f_{wy2}(t) = A_{w2}\sqrt{2gh_2(t)} \approx a_2h_2(t)$$

$$\begin{cases} A_1\dot{h}_1(t) = f_{we}(t) - A_{w1}\sqrt{2gh_1(t)} \\ A_2\dot{h}_2(t) = A_{w1}\sqrt{2gh_1(t)} - A_{w2}\sqrt{2gh_2(t)} \end{cases}$$

$$\begin{cases} A_1\dot{h}_1(t) = f_{we}(t) - a_1h_1(t) \\ A_2\dot{h}_2(t) = a_1h_1(t) - a_2h_2(t) \end{cases}$$



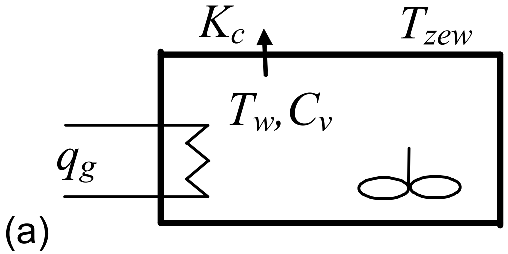
$$\begin{cases} A_1\dot{h}_1(t) = f_{we}(t) - f_{wy1}(t) \\ A_2\dot{h}_2(t) = f_{wy1}(t) - f_{wy2}(t) \end{cases}$$

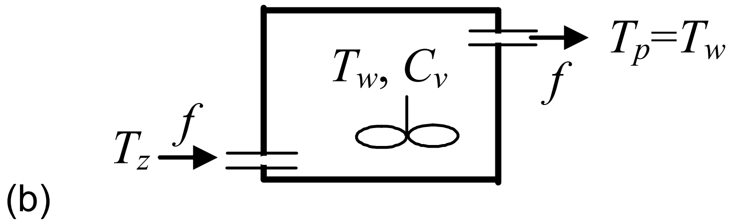
$$f_{wy1}(t) = A_{w1}\sqrt{2g\left(h_1(t) - h_2(t)\right)} \approx a_1\left(h_1(t) - h_2(t)\right)$$

$$f_{wy2}(t) = A_{w2}\sqrt{2gh_2(t)} \approx a_2h_2(t)$$

$$\begin{cases} A_1\dot{h}_1(t) = f_{we}(t) - A_{w1}\sqrt{2g\left(h_1(t) - h_2(t)\right)} \\ A_2\dot{h}_2(t) = A_{w1}\sqrt{2g\left(h_1(t) - h_2(t)\right)} - A_{w2}\sqrt{2gh_2(t)} \end{cases}$$

$$\begin{cases} A_1\dot{h}_1(t) = f_{we}(t) - a_1\left(h_1(t) - h_2(t)\right) \\ A_2\dot{h}_2(t) = a_1\left(h_1(t) - h_2(t)\right) - a_2h_2(t) \end{cases}$$





Założenie o doskonałym mieszaniu

1) Zawartość magazynu $$Q(t) = c_p \rho V T(t) = C_V T(t)$$

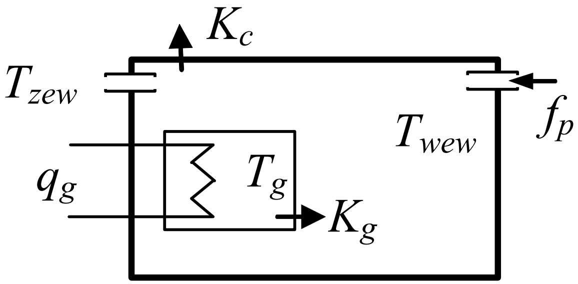
2) Zmiana zawartości magazynu $$\frac{dQ(t)}{dt} = C_V \frac{dT_w(t)}{dt} = C_V \dot{T}_w(t)$$

3) Bilans strumieni wpływających i wypływających [W] $$C_v \dot{T}_w(t) = q_{we}(t) - q_{wy}(t)$$

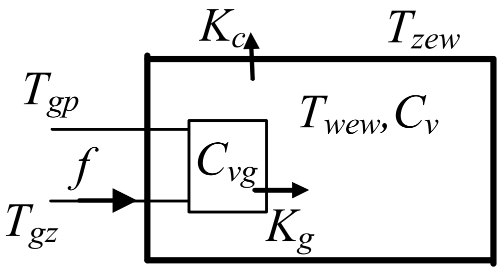
$$C_v \dot{T}_w(t) = q_g(t) - K_c\big(T_w(t) - T_{zew}(t)\big)$$

$$C_v \dot{T}_w(t) = c_p \rho\, f(t) T_z(t) - c_p \rho\, f(t) T_w(t)$$

4) Zmienne wejściowe i wyjściowe, kompletność modelu

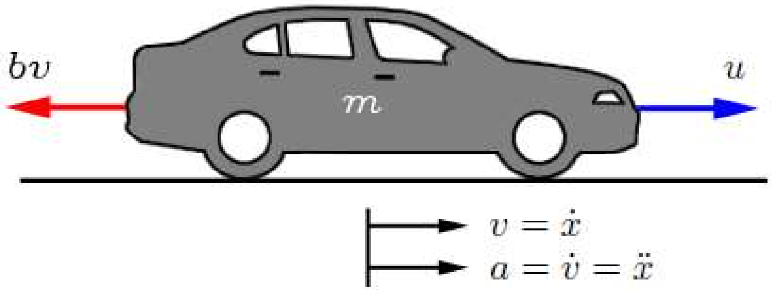


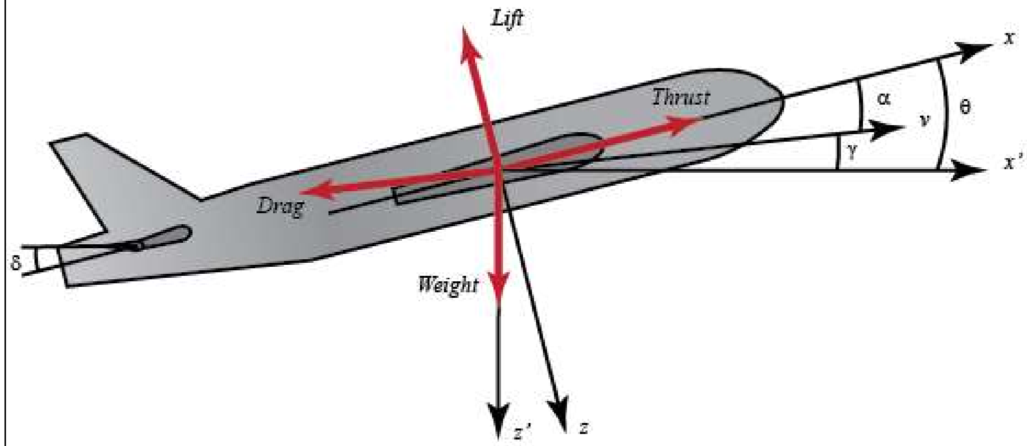
$$\begin{cases} C_{vw}\dot{T}_{wew}(t) = K_g\left(T_g(t) - T_{wew}(t)\right) - K_c\left(T_{wew}(t) - T_{zew}(t)\right) - c_{pp}\rho_p f_p(t)\left(T_{wew}(t) - T_{zew}(t)\right) \\ C_{vg}\dot{T}_g(t) = q_g(t) - K_g\left(T_g(t) - T_{wew}(t)\right) \end{cases}$$

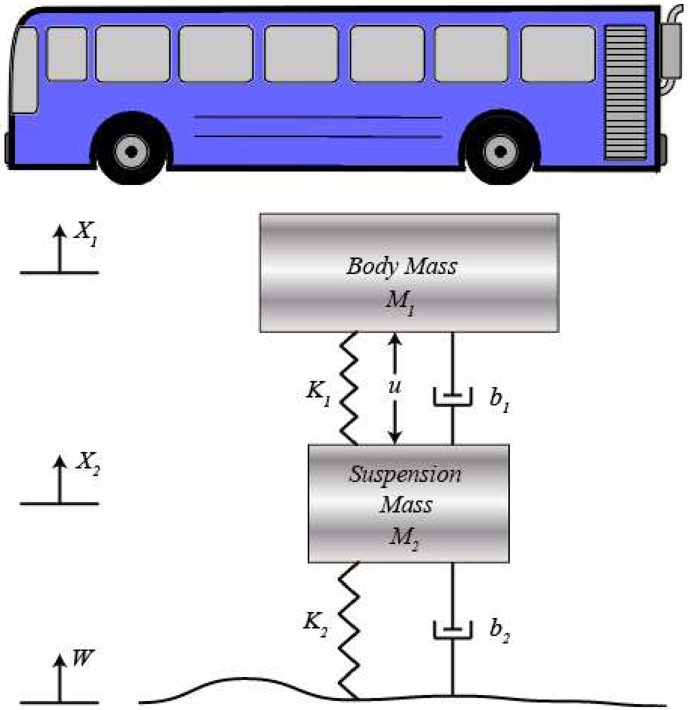


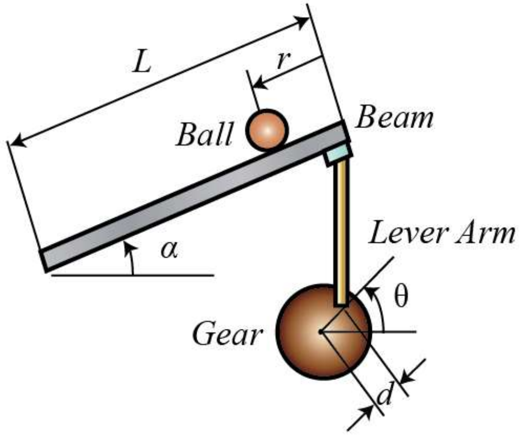
$$\begin{cases} C_{vg}\dot{T}_{gp}(t) = c_{pw}\rho_{pw} f(t) T_{gz}(t) - c_{pw}\rho_{pw} f(t) T_{gp}(t) - K_g\left(T_{gp}(t) - T_{wew}(t)\right) \\ C_{vw}\dot{T}_{wew}(t) = K_g\left(T_{gp}(t) - T_{wew}(t)\right) - K_c\left(T_{wew}(t) - T_{zew}(t)\right) \end{cases}$$

$$\begin{cases} C_{vg}\dot{T}_{gp}(t) = c_{pw}\rho_{pw} f(t)\left(T_{gz}(t) - T_{gp}(t)\right) - K_g\left(T_{gp}(t) - T_{wew}(t)\right) \\ C_{vw}\dot{T}_{wew}(t) = K_g\left(T_{gp}(t) - T_{wew}(t)\right) - K_c\left(T_{wew}(t) - T_{zew}(t)\right) \end{cases}$$



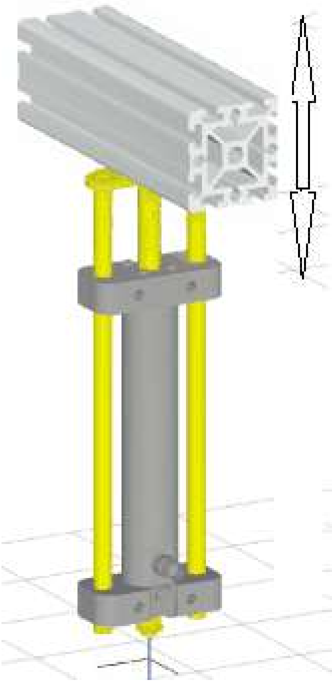


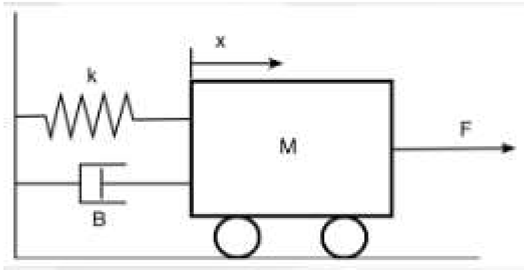


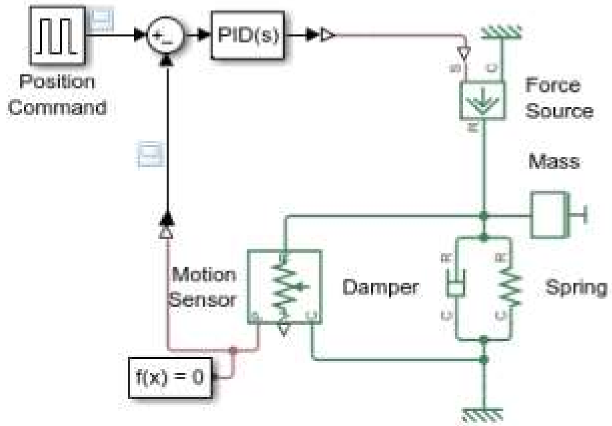


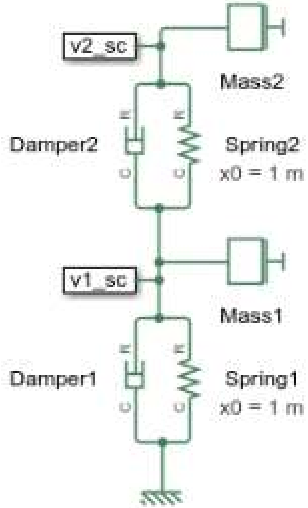
https://ctms.engin.umich.edu/CTMS

model zawieszenia









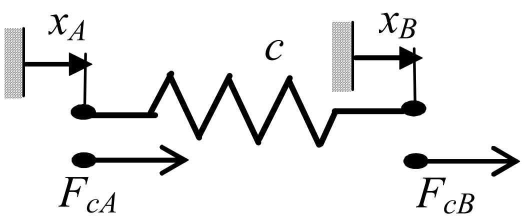
https://simultus.pl/

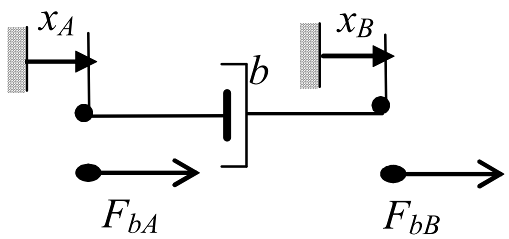
https://simultus.pl/help/modelowanie-uk-adow-dynamicznych-z-przyk-adem-5223#

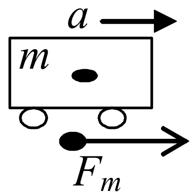
https://www.mathworks.com/products/simscape.html

Założenie – jeden kierunek działania sił

1) Opis działania układu za pomocą idealnych elementów







$$F_{cA}(t) = c\left(x_A(t) - x_B(t)\right)$$

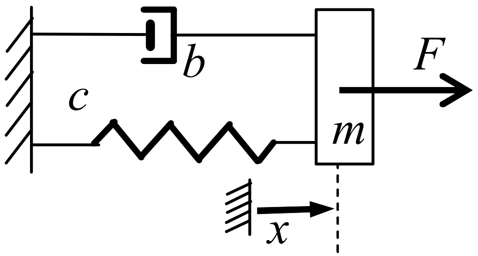
$$F_{cB}(t) = c\left(x_B(t) - x_A(t)\right)$$

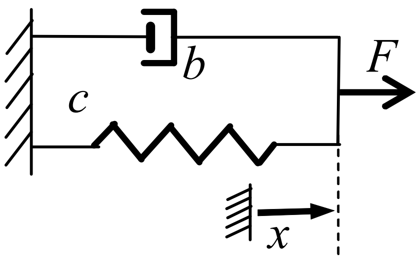
$$F_{bA}(t) = b\left(\dot{x}_A(t) - \dot{x}_B(t)\right)$$

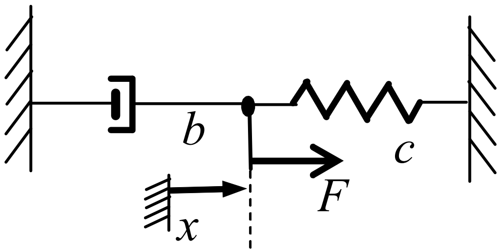
$$F_{bB}(t) = b\left(\dot{x}_B(t) - \dot{x}_A(t)\right)$$

$$F_m(t) = m\ddot{x}(t)$$

2) Punkt bilansowania sił





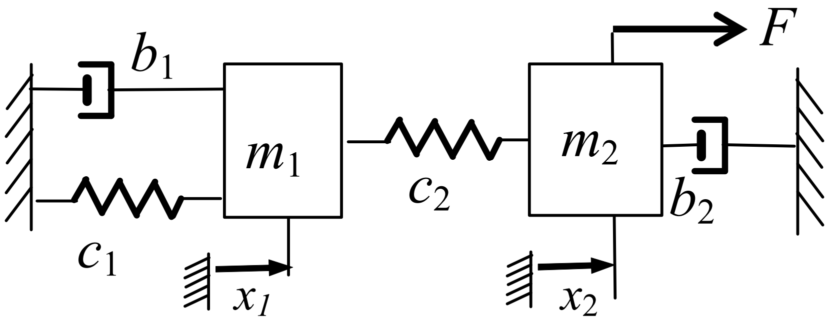


3) Bilans sił [N]

$$m\ddot{x}(t) + b\dot{x}(t) + cx(t) = F(t)$$

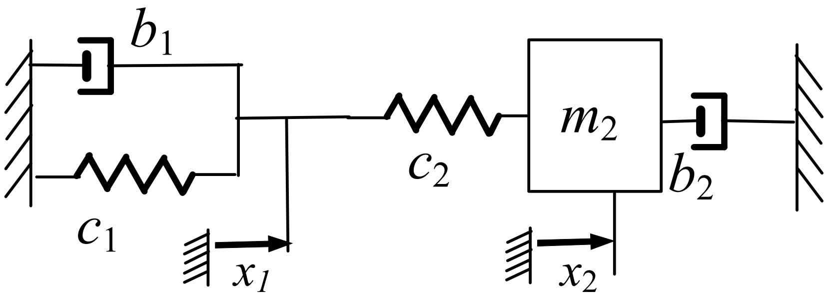
$$b\dot{x}(t) + cx(t) = F(t)$$

$$b\dot{x}(t) + cx(t) = F(t)$$



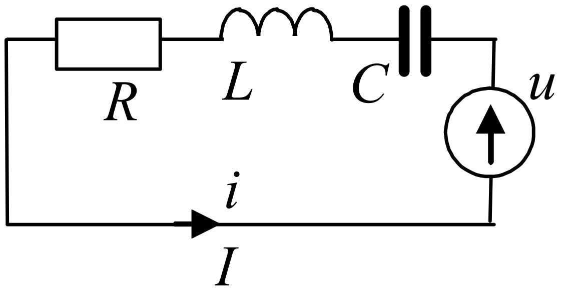
$$\begin{cases} F = m_2\ddot{x}_2 + b_2\dot{x}_2 + c_2(x_2 - x_1) \\ 0 = m_1\ddot{x}_1 + b_1\dot{x}_1 + c_1x_1 + c_2(x_1 - x_2) \end{cases}$$

(2 punkty, 2 masy)



$$\begin{cases} 0 = m_2\ddot{x}_2 + b_2\dot{x}_2 + c_2(x_2 - x_1) \\ 0 = b_1\dot{x}_1 + c_1x_1 + c_2(x_1 - x_2) \end{cases}$$

(2 punkty, 2 masy, bez zewnętrzbej siły)



(1) $$j\omega L\, I + R\, I + \frac{1}{j\omega C} I = U$$

(2) $$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)dt = u(t)$$

(3) $$L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C} q(t) = u(t)$$

(4) $$sL\, i(s) + R\, i(s) + \frac{1}{sC} i(s) = u(s)$$

(5) $$i(s) = \frac{sC}{s^2 LC + sRC + 1} u(s)$$

$$u(t) = U\sin(\omega t)$$

$$i(t) = I\sin(\omega t + \varphi)$$

$$s = j\omega$$

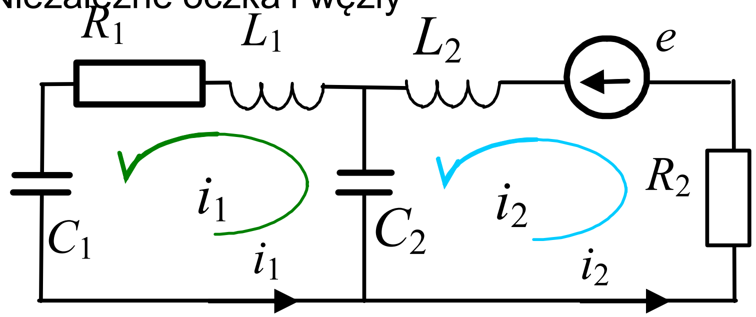
$$i(t) = \frac{dq(t)}{dt}$$

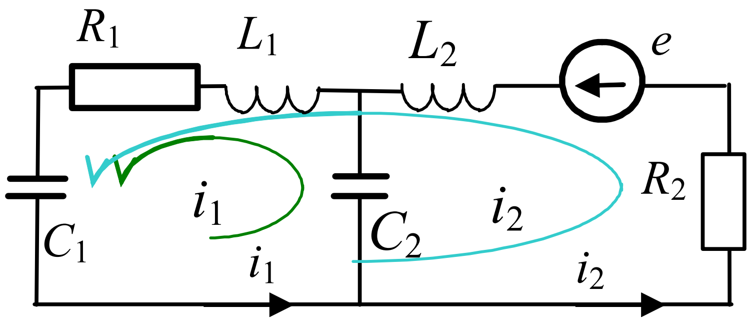
$$i(s) = sq(s)$$

1) Opis działania układu za pomocą idealnych elementów

	Opis napięciowo-prądowy $u(i)$		O.prąd.-napięciowy $i(u)$	$u(q)$	Impedancje $Z(s)$	Impedancje $Z(j\omega)$
rezystor (R)	$u(t)=Ri(t)$	$u(s)=Ri(s)$	$i(t)=Gu(t)$	$u(t)=R\dot{q}(t)$	R	R
kondensator (C)	$u(t)=\frac{1}{C}\int i(t)dt$	$u(s)=\frac{1}{sC}i(s)$	$i(t)=C\frac{du(t)}{dt}$	$u(t)=\frac{1}{C}q(t)$	$\frac{1}{sC}$	$\frac{1}{j\omega C}$
cewka (L)	$e_L(t)=-L\frac{di(t)}{dt}$	$u(s)=sLi(s)$	$i(t)=\frac{1}{L}\int u(t)dt$	$u(t)=L\ddot{q}(t)$	sL	$j\omega L$

2) Niezależne oczka i węzły





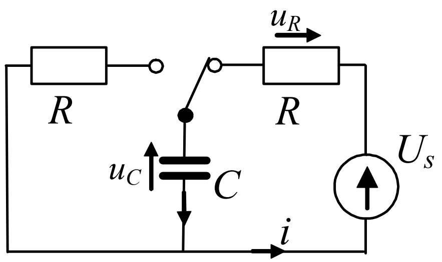
3) Bilans napięć w oczkach obwodu i/lub prądów (ładunków) w węzach [V, A]

$$\begin{cases} e = sL_2 i_2 + R_2 i_2(s) + \dfrac{i_2 - i_1}{sC_2} \\ 0 = sL_1 i_1 + R_1 i_1 + \dfrac{i_1}{sC_1} + \dfrac{i_1 - i_2}{sC_2} \end{cases}$$

$$\begin{cases} e = L_2 \dfrac{di_2}{dt} + R_2 i_2 + \int \dfrac{i_2 - i_1}{C_2} dt \\ 0 = L_1 \dfrac{di_1}{dt} + R_1 i_1 + \int \dfrac{i_1}{C_1} dt + \int \dfrac{i_1 - i_2}{C_2} dt \end{cases}$$

$$\begin{cases} e = L_2 \ddot{q}_2 + R_2 \dot{q}_2 + \dfrac{q_2 - q_1}{C_2} \\ 0 = L_1 \ddot{q}_1 + R_1 \dot{q}_1 + \dfrac{q_1}{C_1} + \dfrac{q_1 - q_2}{C_2} \end{cases}$$

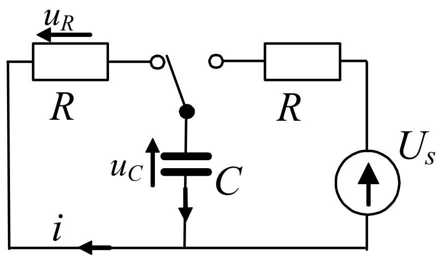
Ładowanie/rozładowanie kondensatora



$$u_R(t)+u_C(t)=U_s$$

$$Ri(t)+\frac{q(t)}{C}=U_s$$

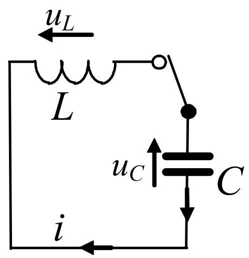
$$R\dot{q}(t)+\frac{1}{C}q(t)=U_s \quad , \; q(0)=0$$



$$u_R(t)+u_C(t)=0$$

$$Ri(t)+\frac{q(t)}{C}=0$$

$$R\dot{q}(t)+\frac{1}{C}q(t)=0 \quad , \; q(0)=q_{\max}=CU_s$$



$$u_L(t)+u_C(t)=0$$

$$L\frac{di(t)}{dt}+\frac{1}{C}\int i(t)dt=0$$

$$L\ddot{q}(t)+\frac{1}{C}q(t)=0$$

$$\ddot{q}(t)+\frac{1}{LC}q(t)=0$$

$$\omega=\sqrt{\frac{1}{LC}}$$

r.s.) $R\lambda+\frac{1}{C}=0 \;\rightarrow\; \lambda=-\frac{1}{RC}$

	Ładowanie	Rozładowanie
r.w.)	$\frac{1}{C}q(t)=U_s$ $q_w(t)=CU_s=q_{\max}$	$\frac{1}{C}q(t)=0$ $q_w(t)=0$
r.o.)	$q(t)=Ae^{-\frac{1}{RC}t}+CU_s$	$q(t)=Ae^{-\frac{1}{RC}t}$
w.p.)	$0=Ae^{-\frac{1}{RC}0}+CU_s \;\rightarrow\; A=-CU_s$	$CU_s=Ae^{-\frac{1}{RC}0}{}_s \;\rightarrow\; A=CU_s$
r.s.)	$q(t)=CU_s\left(1-e^{-\frac{1}{RC}t}\right)$	$q(t)=CU_se^{-\frac{1}{RC}t}$

$$i(t)=\frac{dq(t)}{dt}=\frac{U_s}{R}e^{-\frac{1}{RC}t} \quad , \; u_C(t)=\frac{q(t)}{C}=U_s\left(1-e^{-\frac{1}{RC}t}\right)$$

$$i(t)=\frac{dq(t)}{dt}=-\frac{U_s}{R}e^{-\frac{1}{RC}t} \quad , \; u_C(t)=\frac{q(t)}{C}=U_se^{-\frac{1}{RC}t}$$